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This curriculum bulletin is designed to help teachers meet the diverse needs in mathematics of the children in fifth grade classes. In addition to the emphasis that is placed on arithmetic computational skills, the bulletin shows how to include other areas considered important, such as concepts, skills, and ideas from algebra and geometry. The 80 units of the bulletin are organized into the following categories: (a) sets, number, numeration; (b) operations; and (c) geometry and measurement. The units are sequentially planned and follow a spiral pattern. (RP)

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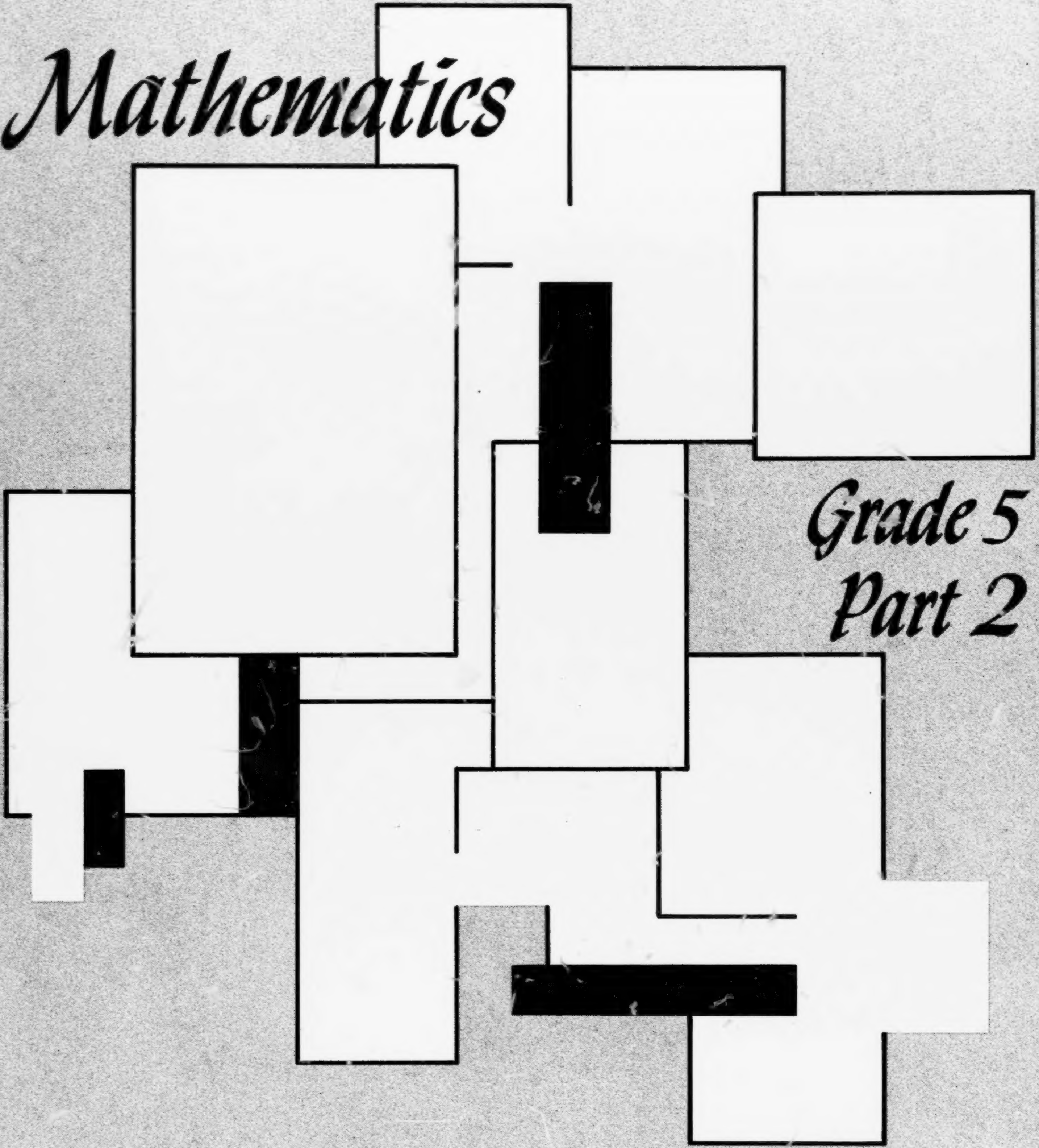
Mathematics



Grade 5
Part 2

ED023600

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Board of Education of the City of New York

NOTE TO THE TEACHER

Mathematics Grade 5 **is presented in two sections. This is the second of the two parts. It contains Units 43 through 80.**

Units 1 through 42 were published in *Mathematics Grade 5 Part I.*

For your convenience the Scope and Sequence following this page also includes those units developed in Part I.

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Number 5a

Mathematics
Grade 5
Part 2

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FOREWORD

The MATHEMATICS, GRADE 5, CURRICULUM BULLETIN is one of a planned series of bulletins designed to meet the needs of teachers and supervisors who are working to improve the achievement level of mathematics in our schools. The material has been planned to help teachers meet the diverse mathematical needs of the children in fifth-grade classes in our schools. In addition to the emphasis that is always placed on arithmetic computational skills, this bulletin shows how to include other areas considered important, such as, concepts, skills, and ideas from Algebra and Geometry.

Use of the new bulletin will provide articulation with grade 4 mathematics and with the mathematics of grades 6, 7, and 8. The publication completes a four-year sequence in Intermediate School Mathematics based on the newer mathematical philosophy of what should and can be taught to children in grades 5 through 8. Revisions will be made in this publication as a result of use in schools during the introductory period.

ACKNOWLEDGMENTS

The impetus for Mathematics Grade 5 was provided by Mr. George Grossman, Acting Director of the Bureau of Mathematics and by Mrs. Ella Simpson, Acting Assistant Director of the Bureau of Mathematics who recognized the need for a more modern approach to the teaching of mathematics in Grade 5.

Dr. Joseph O. Loretan,* Deputy Superintendent of Schools and Mrs. Helene Lloyd, Assistant Superintendent supervised the project.

Assistant Superintendent Dr. William Bristow, Director of The Bureau of Curriculum Research and members of his Bureau cooperated in the initial discussions.

Mrs. Leona Critchley and Mrs. Alice Lombardi, Staff Coordinators of the Bureau of Mathematics, planned and prepared the sequence and organization, wrote the units, and edited the final draft for publication.

Mrs. Eva Pollack, Staff Coordinator of the Bureau of Mathematics, reviewed the units, made suggestions, and rewrote parts of some units.

Mr. Grossman, as Acting Director of Mathematics edited and suggested revisions and additions.

Special thanks are given to Mrs. Ella Simpson, Acting Assistant Director, for her encouragement and advice.

The constructive criticism of Mrs. Jeanette Eisner, Mrs. Blanche Gladstone, Mr. Leonard Simon and Mr. Frank Wohlfort is gratefully appreciated.

Mrs. Gertrude Fischer, Mrs. Marilyn Katz, Mrs. Thelma Morris, and Mr. Eugene Erdos typed the manuscript. Gratitude is expressed to Dr. Charles Warshauer who reviewed the manuscript for English.

Miss Anne Piccini prepared the diagrams and designed the cover. Mr. Albert Lacerre and Mr. Maurice Basseches facilitated the printing.

Grateful acknowledgment is made to all those responsible for the production of Mathematics Cycles Grade 5 and Mathematics Cycles Grade 6.

******Deceased***

INTRODUCTION

This is the second part of a two part bulletin that has been prepared as a revision of the earlier Mathematics 5 Cycles. It includes all of the topics in Mathematics Cycles — Grade 5(1) enriched and expanded to include the newer emphasis of a modern mathematics curriculum as found in the 1965 edition of the Mathematics 6 Bulletin. All relevant material in these bulletins has been utilized.

This will mean that students completing this course will be in a better position to complete a more thorough course in Mathematics 6.

It *is* important for children to develop speed and accuracy in computation, but in addition to computational skills, it is important for children to develop understanding not only of arithmetic concepts but of concepts from algebra and geometry.

The 80 units of this bulletin are organized into 3 categories:

Sets; Number; Numeration
Operations
Geometry and Measurement

These categories are shown in the Scope and Sequence Chart on page vi to xi. This chart may also be considered a Table of Contents.

The 80 units are sequentially planned. For example: After an introduction to sets, under the category "Sets," children are led to see the relation between Union of Sets and Addition of Whole Numbers under the category "Operations." This pattern is followed throughout.

The units also follow a *spiral* pattern, in that development of concepts and operations are repeated at increasing levels of understanding.

Concepts from Algebra, such as: open sentences, relations between numbers, graphing of solution sets, are included in the exercises of most of the units. Concepts from geometry are also included. A "Note to Teacher" is included in those units where it was felt the teacher might want further clarification of the mathematical concepts connected with the unit and/or to understand reasons for the developmental material.

Objectives for each unit are clearly stated immediately before the "Teaching Suggestions" designed for the implementation of those objectives.

Review of background necessary for the introduction of new topics is suggested where necessary. For example: Before adding and subtracting fractional numbers using the least common denominator method, suggestions are made for renaming fractions, regrouping fractions, etc.

Asterisks before a unit or before an item within that unit indicate that these developments may be used at the discretion of the teacher.

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(1) Mathematics Cycles, Grades 5 — Curriculum Bulletin. 1961-62 Series, No. 6

SCOPE AND SEQUENCE

Pages vi to xi

Grade 5

SCOPE AND SEQUENCE

* — indicates optional units
page numbers below — indicate location of units

SETS; NUMBER; NUMERATION	OPERATIONS	GEOMETRY AND MEASUREMENT
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2. Sets: One-to-one Correspondence; Equivalent Sets; Number as a Property of; Set of Counting Numbers; Equalities and Inequalities p. 8		
3. Set of Whole Numbers: Concept of Number and Numeral; Reading and Writing Numerals p. 16		
		4. Geometry: Set of Points; Curves; Number Line p. 22
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	6. Addition Related to Sets; Open, True, False Sentences; Replacement Set, Truth Set p. 36	
7. Subsets p. 47		
	8. Subtraction Related to Sets; Mathematical Sentences p. 52	
9. Systems of Numeration; Base 10 Expanded Notation p. 65		
10. Set of Fractional Numbers: Concepts and Comparisons p. 69		
		11. Measurement: Concepts of Length p. 81
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13. Systems of Numeration: Roman System p. 91		
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	15. Multiplication of Whole Numbers: Facts and Extensions of Facts; Arrays; Factors; Properties Applied p. 99	
	16. Division of Whole Numbers: Facts and Extensions of Facts; Related to "Finding the Missing Factor" p. 121	
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	18. Addition and Subtraction of Whole Numbers: Properties Applied; Vertical Format p. 134	
	19. Addition of Fractional Numbers: Properties Applied; Open Sentences; Placeholders p. 140	
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50. Set of Fractional Numbers: Sixteenths; Locating Other Fractions p. 348		
		51. Measurement: Length; Fractional Parts of the Inch; Approximate Measurements; Scale Drawing p. 355
52. Number System: "Clock" Arithmetic p. 361		
*53. Number: Modular System p. 367		
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	55. Multiplication of Whole Numbers: Exploring Patterns p. 378	

SCOPE AND SEQUENCE

* — indicates optional units
page numbers below — indicate location of units

SETS; NUMBER; NUMERATION	OPERATIONS	GEOMETRY AND MEASUREMENT
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	57. Division of Whole Numbers: Property of "1"; Property of Zero p. 389	
58. Set of Fractional Numbers: Concepts of Fifths and Tenths; Common Fractional Form p. 395		
59. Set of Fractional Numbers: Tenths; Decimal Form p. 403		
	60. Set of Fractional Numbers: Adding and Subtracting Tenths; Decimal Form; Horizontal and Vertical Format; Without Exchange p. 414	
61. Extended Understanding of Sets: Intersection of Sets p. 421		
62. Set of Fractional Numbers: Finding Greatest Common Factor of Two Numbers; Common Denominator; Least Common Denominator p. 425		
	63. Set of Fractional Numbers: Addition and Subtraction; Common Form; Horizontal and Vertical Format p. 433	
	64. Set of Fractional Numbers: Adding and Subtracting Tenths; Decimal Forms; With Exchange p. 440	
		65. Geometry: Circle p. 445
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	67. Statistics: Finding Average p. 459	
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	70. Set of Fractional Numbers: Adding and Subtracting Hundredths; Decimal Form p. 481	
	71. Set of Fractional Numbers: Common Form; Multiplication; Properties Applied p. 487	
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GEOMETRY AND MEASUREMENT

UNIT 43 - MEASUREMENT: WEIGHT

Objectives: To help children find number of ounces in $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ pound

To help children find weight that cannot be measured directly.

To introduce concept of Ton

To help children develop tables of relationships

To provide practice in changing to larger and to smaller units.

TEACHING SUGGESTIONS

1. Reinforce number of ounces in 1 pound.

2. Extend development to include number of ounces in $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ pound.

The following are suggested experience situations involving weights:

Estimating quantities and cost of refreshments needed for a party.

Sharing quantities and costs among pupils.
Problem situation: Buying candy for the party.
Chicken corn is sold in 4 ounce bags or by the pound.
How shall we buy it? Why?

Keeping individual records to show weight in pounds and fractional parts of pounds. Then, noting gain or loss in weight.

Discussing required postage in relation to weight of mail. Referring to printed table of Post Office for information about rates of 1st class mail, 2nd class mail, air mail, etc. Developing problems based on this information.

Listing items bought by the ounce of $\frac{1}{4}$ pound

Discussing reasons for purchasing small quantities of spices, dried mushrooms, grated cheese, shelled nuts.

Considering size of family when planning menus.

Comparing prices with weight to determine best value.

3. Developing ways of finding weight that cannot be measured directly.

For a baby or pet: weigh an adult holding baby or pet. Weigh adult. Deduct weight of adult.

For contents of a jar or pan: weigh empty jar, then filled jar. Deduct weight of empty jar.

4. Suggested practice exercises

a. A cake recipe requires $1\frac{1}{4}$ pounds of butter. Mother wishes to make double the amount. She will need ____ pounds or ______ ounces.

b. Mark the following True or False.
If false, correct the statement.

$2\frac{3}{4}$ lb. = 44 oz.

8 oz. = $\frac{1}{4}$ lb.

$1\frac{1}{8}$ oz. $>$ 18 oz.

c. Complete the following:

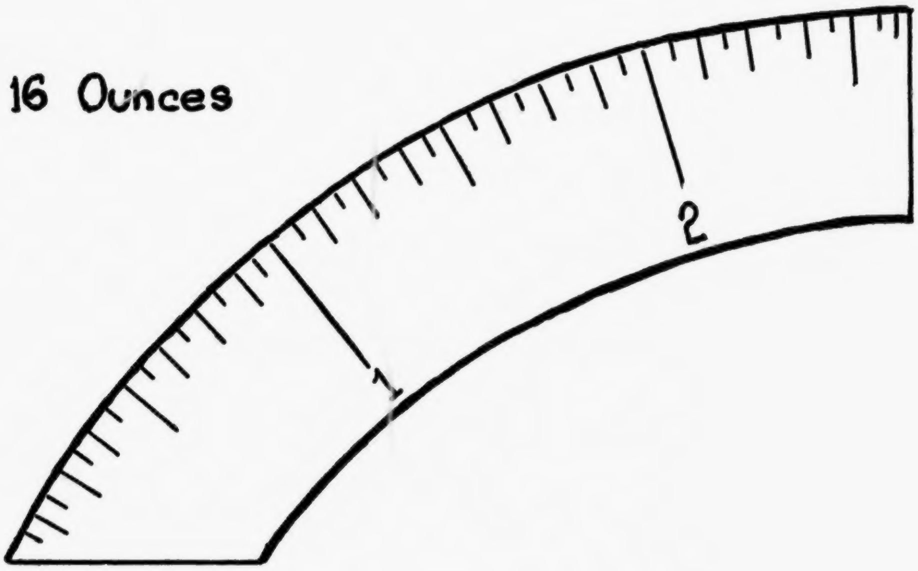
$2\frac{1}{2}$ lb. = ☐ oz.

16 oz. = $\frac{\square}{8}$ lb.

The number of ounces in $\frac{1}{4}$ lb. is _____.

5. Collect pictures of dials or portions of dials of weighing devices. Have children explain how the device is used and read. Include a hanging spring scale, bathroom scale and equal arm balances.

Construct a model of the dial on a meat scale. Divide each pound interval to show ounce divisions.



6. Introduce unit of Ton (2000 lbs.)

Suggested Experiences

Family Car	Discuss weight in relation to license fee.
Trucks	Capacity is measured in tons. Have children look at the side of trucks and report findings to class.
Elevators	Notices in elevators state limit of weight in terms of ton. Estimate the average weight per person before they find the number of people who can safely use an elevator.
Bridges	Limits of weight are posted at the terminals.

Reinforce equivalents

Reinforce equivalents between ounces and pounds; pounds and tons: Experience situations involving change to smaller units.

Purchasing packaged and canned goods; comparing quantities to aid in determining the more economical purchases.

7. Develop tables of relationships.

Tables of relationships may be prepared by the children.

Fractional Parts of Pounds Expressed in Ounces

1 lb. = ☐ oz.

$\frac{1}{2}$ lb. = ☐ oz.

$\frac{1}{4}$ lb. = ☐ oz.

$\frac{1}{8}$ lb. = ☐ oz.

$\frac{3}{4}$ lb. = ☐ oz.

Fractional Parts of the Ton Expressed in Pounds

1 ton = ☐ lb.

$\frac{1}{2}$ ton = ☐ lb.

$\frac{1}{4}$ ton = ☐ lb.

8. Provide practice in changing to larger and smaller units.

20 oz. = 1 lb. ☐ oz. or $1\frac{1}{4}$ lb.

3000 lb. = 1 ton ☐ lb. or $1\frac{1}{2}$ tons

$1\frac{3}{4}$ lb. = 16 oz. and ☐ oz. = ☐ oz.

$2\frac{1}{4}$ ton = 4000 lb. and ☐ lb. = ☐ lb.

9. Discuss letters and numbers on the side of trucks in New York State and their meaning.

NYUW - New York Unladen Weight
NYML - New York Maximum Load
NYMGW - New York Maximum Gross Weight

Devise problems to find the net weight of a load on a truck.

EVALUATION and / or PRACTICE

SUGGESTED PROBLEMS

1. Candy is sometimes sold in 8 oz. bags. The storekeeper bought 25 lb. can of candy. How many 8 oz. bags can be filled?

2. The cost of sending a 1 ounce letter or package by First Class Mail is 5 cents for 1 ounce. John mailed a small package which weighed $\frac{1}{4}$ lbs. How much did he pay for stamps?

3. A 2 lb. box is half full of sugar. How many ounces of sugar are in the box?

4. Pick up one dozen new pencils. About how much do you think they weigh?

   1 lb. $\frac{1}{4}$ lb.   3 oz.   8 oz.

   How can you check the estimate?

5. The clerk in the Post Office weighed a package for Mrs. White. She sent it by First Class Mail and had to pay 40 cents. Rate: 5 cents per oz.

   a. Did the package weigh more or less than 1 lb.?
   b. How many ounces did the package weigh?

6. A box of mustard is labeled, Contents, 2 oz. The mustard weighs what part of a pound?

7. A bale of cotton weighs $\frac{1}{4}$ ton. A truck carried 48 bales to a ship.

   a. How many tons were on the truck?
   b. How many pounds did each bale weigh?

8. Two ounces of meat are put into each sandwich at the "Sandwich Shop".

   a. How many sandwiches can be made from 1 pound of meat?
   b. How many sandwiches can be made from 1 pound 6 ounces?
   c. How many pounds are needed to make 2 dozen sandwiches?

9. Is it safe for an empty truck weighing 2500 pounds to cross a bridge when the sign reads, "Limit 3 Tons"? Yes? No? Why?

10. A truck marked "Capacity 3 Tons" can carry a load weighing . . . . pounds.

11. It is safe for an elevator in a Department Store to carry a load of a $\frac{1}{2}$ ton. How many children, each weighing about 50 pounds, can ride in it at one time?

12. Additional problems may be found in textbooks.

GEOMETRY AND MEASUREMENT

UNIT 44 - MEASUREMENT: TIME

Objectives: To extend understanding of periods of calendar time
To introduce concept of seconds in a minute
To organize tables to show relationships
To introduce concept of time zones

TEACHING SUGGESTIONS

Clock Time

1. Discuss day's activities done by the clock.

2. Introduce need for seconds as a unit of measure

   Timing athletic events
   Taking pictures, developing pictures
   Testing pulse reading
   Television programs

3. Introduce concept of seconds in a minute.
   Use a watch and a stop watch.

4. Organize tables of time in various ways. For example;

   60 sec. = 1 ______
   60 min. = 1 ______
   24 hr. = 1 ______
   7 days = 1 ______
   52 weeks = 1 ______

5. Reinforce the relationships between units of time and fractional parts of these units, and develop other tables. For example;

   60 min. = ______ hr.
   30 min. = ______ hr.
   15 min. = ______ hr.
   45 min. = ______ hr. etc.

6. Discuss the meaning of an 8 hour day; a 40 hour week; overtime; etc.

* 7. Have children explore: **(Optional)**

Naval Observatory Time

Timekeeping through the ages:
History of telling time by sandglass, time candle,
position of the sun, shadows, water clocks, sundial,
Twenty-four Hour Clock, Ship's Bells

8. Suggested exercises and problems

a. Underline the unit of measure which you would use for each of the following:

Sue wants to time the baking of a cake

minutes seconds days

Measuring the time for the 100 yard dash

seconds hours minutes and seconds

b. Insert the correct symbol, > or < :

45 min. ____ $\frac{1}{4}$ hr.

$\frac{1}{3}$ min. ____ 15 sec.

$\frac{1}{4}$ hr. ____ 25 min.

c. Arrange these measures from longest time to shortest time:

a minute 70 seconds an hour a day a half minute
a second a half hour 20 hours 25 hours

d. Fill in the blanks:

$2\frac{1}{2}$ hr. = ____ min. $\frac{1}{2}$ day = ____ hr.

$\frac{1}{4}$ hr. = ____ min. 30 sec. = ____ min.

3 min. = ____ sec. 45 min. = ____ hr.

120 sec. = ____ min. 80 min. = ____ hr.

* e. A ship sailed from New York at 2 P.M. on Monday and docked in London 5 days later at 12 Noon. How many hours did the trip take?

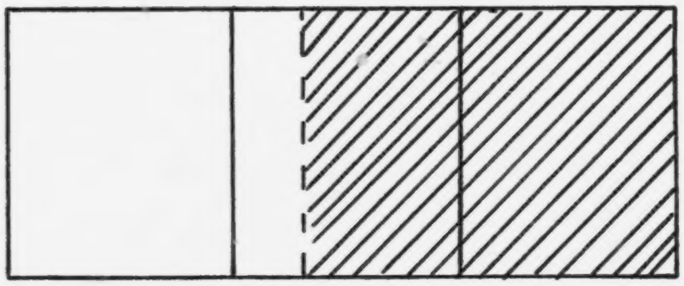
f. Mark the following statements true (T) or false (F).

There are 50 seconds in a minute.
75 minutes = 1 hr. 15 min. = $1\frac{1}{2}$ hr.
10 minutes = $\frac{1}{6}$ hr.
6:45 P.M. to 9:15 P.M. is $2\frac{1}{2}$ hrs.

g. Draw a line from the time recorded in Column A to the words which have the same meaning in Column B.

Column A	Column B
9:45	10 minutes after 11
6:50	10 minutes to 3
2:50	half past 3
7:15	50 minutes after 6
11:10	a quarter past 7
3:30	a quarter to 10

h. This whole bar is drawn to show 3 hours.



Which of the following does the shaded part of the bar show?

about 45 min.
about 1 hour and 45 min.
about 1 hour and 15 min.

Calendar Time

Concept of Leap Year

1. Reinforce number of days in a year.

   Tell children it takes the earth 365 days, 5 hours, 48 minutes, and 49 seconds to make its way around the sun so the length of one year is not exactly 365 days. (This is called a Solar Year.)

   Ask children:

   > **Since the year is considered to consist of 365 days what adjustment is made for the 5 hours, 48 minutes and 49 seconds of the Solar Year?**
   >
   > Approximately what part of a day is this additional time?
   >
   > [$\frac{1}{4}$ of a day]
   >
   > How many years will it take until enough time has accumulated to make another day?
   >
   > Can you tell how our calendar is adjusted to take care of this time?

   Tell children that correction is made on our calendar every fourth year by adding one day to the month of February. This fourth year is called Leap Year and has 366 days.

*2. Children may explore: **(Optional)**

   Various types of calendars

   World, Gregorian, Hebrew, Chinese, Mayan

3. Have children investigate meaning of: decade, score, century, half century, centennial year, sesquicentennial year, generation.

   Relate these terms to time periods in Social Studies, Art, Music, etc. For example, How long ago did the Westward Movement occur?

   Strengthen meaning of terms by constructing various Time Lines:
   - Individual Time Line of child's life. Include ideas of decade.
   - Extend Time Line by decades to a century.
   - Divide centuries into half centuries and decades.
   - Refer to Social Studies textbooks for suggested situations for Time Lines.

4. Suggested problems for practice:

a. Arrange these measures from shortest time to longest time:

17 days 14 years a season 2 months

3 weeks 2 decades a century

b. What did Abraham Lincoln mean when he said, "Four score and seven years ago" ?

c. In what century:

Do we live?

Did the American Revolution take place?

Did Columbus discover New World?

Did Julius Caesar live?

d. The first Thanksgiving Day took place in 1621.
In what century was that?
How many years ago did that happen?

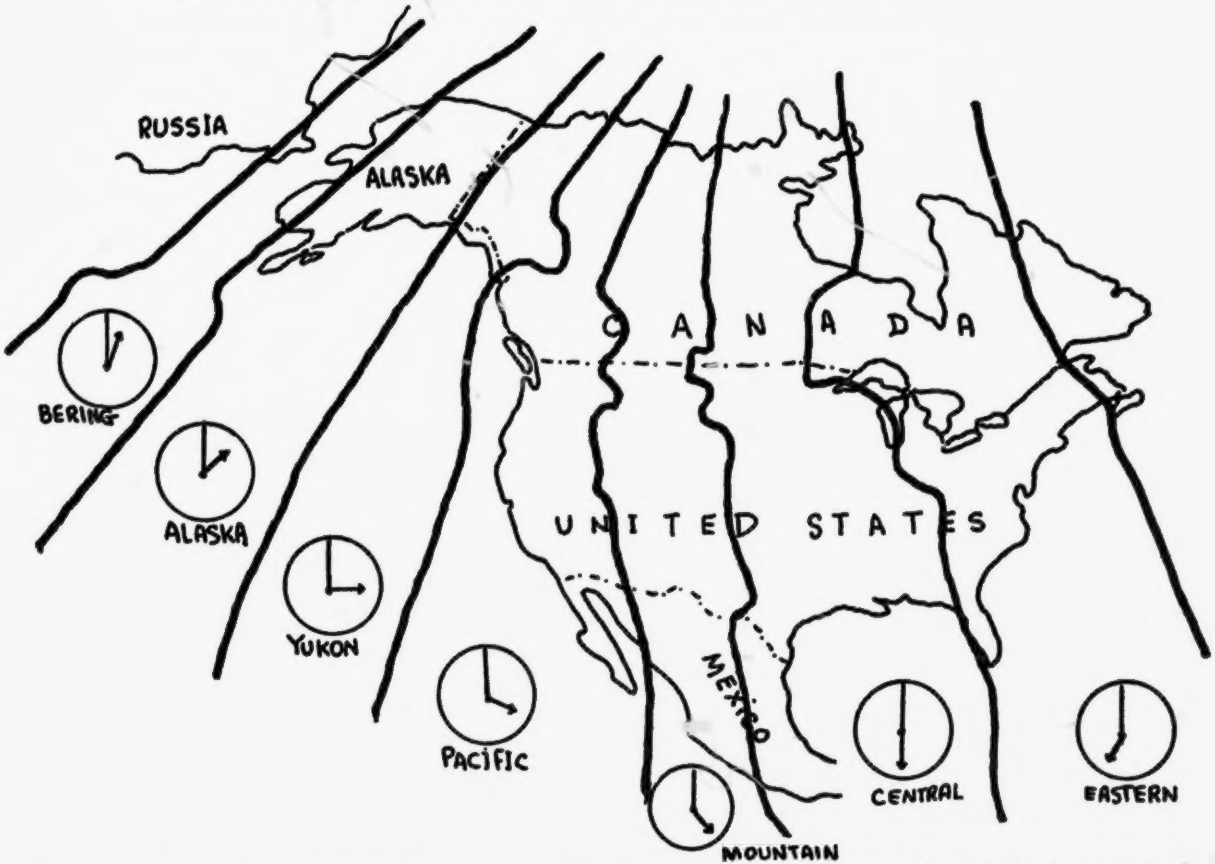
The Star Spangled Banner was written in 1814.
In what century was it written?

Betsy Ross made our flag in 1777.
What century was that?
Which half of the century?

e. How many scores of years are there before the next century?
On what date will the twenty-first century begin?
In what century did the year 34 A.D. occur?

Time Zones in the United States

Discuss time zones as shown in the map below.


RUSSIA
ALASKA
CANADA
UNITED STATES
MEXICO
BERING
ALASKA
YUKON
PACIFIC
MOUNTAIN
CENTRAL
EASTERN

1. In how many time zones are the 50 United States of America? [7]

2. As you travel from New York to San Francisco do you move your watch ahead or back as you enter each time zone? Why?

3. A Television program shown at 7:00 P.M. in Denver will be seen in New York at what time?

   [5:00 P.M.]

4. When it is noon in the State of Hawaii, what time is it in the State of Pennsylvania?

   [5:00 P.M.]

5. Consult newspaper on any given day for time of sunrise in New York to discuss the following:

   When it is sunrise in New York, what time is it in Alaska?

6. Refer to the map above to find whether the United States shares time zones with any other countries.

SETS; NUMBER; NUMERATION

UNIT 45 - NUMERATION: ROMAN SYSTEM

NOTE TO TEACHER

Continued study of the Roman System of Numeration will reinforce an understanding of the Decimal System of Numeration, will emphasize the efficiency of the Decimal System of Numeration and will reinforce the distinction between number and numeral.

Objective: To compare Roman and Hindu-Arabic Systems of Numeration

TEACHING SUGGESTIONS

1. Test children's ability to translate Roman numerals into Arabic numerals and the reverse, for numbers through 500.

2. Introduce M as the Roman numeral representing 1000. Have the children derive the Roman numeral for 900 by using the subtractive principle: CM = 100 less than 1000.

3. Reinforce the principles used to combine the seven symbols: I, V, X, L, D, C, M

The Rule of Addition:	VI, LX, DC, etc.
The Rule of Subtraction:	IV, XL, CD, etc.
The Rule of Repetition:	III, XXX, CCC, etc. (In later Roman Notation, a symbol is repeated no more than 3 times)

Symbols which cannot be repeated: V because 2 fives = X;
L because 2 fifties = C;
D because 2 five hundreds = M.

Have children explain why we say the Roman System of Numeration involves "Repetition"; "Addition"; "Subtraction". Do any of these principles apply to The Hindu-Arabic System?

4. Children may construct a chart to compare numbers represented by Roman numerals and Hindu-Arabic numerals.

ONES:	1	2	3	4	5	6	7	8	9
	I	II	III	IV	V	VI	VII	VIII	IX
TENS:	10	20	30	40	50	60	70	80	90
	X	XX	XXX	XL	L	LX	LXX	LXXX	XC
HUNDREDS:	100	200	300	400	500	600	700	800	900
	C	CC	CCC	CD	D	DC	DCC	DCCC	CM
THOUSANDS:	1000								
	M								

5. Children may construct a chart to make comparisons between systems of numeration. For example,

Hindu Arabic	Roman
Number of Symbols: 10	Number of Symbols: 7
1, 2, 3, 4, 5, 6, 7, 8, 9, 0	I V X L C D M
Symbol "0" indicates a zero number of ones, tens, hundreds, etc.	No symbol for "zero" number of I's, V's, X's, etc.
Notation	**Notation**
System - simple	System - complicated
7 70 700 — Two digits needed to write all three numerals	VII LXX DCC — Six letters needed to write all three numerals
888 — 1 digit needed	DCCCLXXXVIII — 12 letters needed

(chart continued)

Base: 10	Base: 10
10 = 10 x 1 100 = 10 x 10 1000 = 10 x 10 x 10	X = 10 x 1 C = 10 x 10 M = 10 x 10 x 10 For convenience, symbols for midway points are used. I. . . V . . . X I. . . L . . . C I. . . D . . . M
Principles	**Principles**
Addition Place Value	Repetition Addition Subtraction

Have children refer to the chart to:

Compare the number of symbols used in the Roman System (7) with the number of symbols used in the Decimal System. (10)
Tell how the use of the symbol for zero makes the Decimal System more efficient.
Think out relative values (ratios) of the basic symbols in the Roman System;
2 to 1, 5 to 1, etc.

M = (2 x D) D = (5 x C) C = (2 x L) L = (5 x X)
X = (2 x V) V = (5 x 1) I = 1

Compare the principles of the Roman System with those of the Hindu-Arabic System. They discuss the advantages of our system of numeration.

6. Children should make the following comparisons:

Hindu Arabic System of Numeration	Roman System of Numeration
Relative Values	**Relative Values**
The relative value represented by a particular digit is 10 times the value of the same digit to its right.	The relative value represented by the basic letters are 5 to 1, 2 to 1, 5 to 1, 2 to 1, 5 to 1, and 2 to 1.

(Relative values continued)

Thous.	Hun.	Tens	Ones
1	1	1	1

V = 5 times I
X = 2 times V
L = 5 times X
C = 2 times L
D = 5 times C
M = 2 times D

Computation is Simple

	Tens	Ones
	3	7
+	5	8
	9	5

$$\begin{array}{r} 2\ 3 \\ \underline{\text{x}\ \ 3} \\ 6\ 9 \end{array}$$

Computation is Complicated

Fifties	Tens	Fives	Ones	
	XXX	V	II	
L		V	III	
L	XXX	VV	IIIII	=

LXXXXV = XCV

Fifties	Tens	Fives	Ones	
	XX		III	23
			III	x 3
	XX		III	(1 x 23)
	XX		III	(1 x 23)
	XX		III	(1 x 23)
	XXXXXX		IIIIIIIII	
L	X		IX	

Ask children:

a. In the Base 10 System upon what does the value of each digit in the numeral 444 depend?

b. How would 444 be written in the Roman System?

c. What is the sum of the values of each basic symbol in LXVI ?

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Write the following as Roman Numerals: 39; 44; 279; 500; 177.

2. Write the following as Arabic Numerals:

LXXII; XLV; LIII; MC; MMCD.

3. Write the Roman Numerals counting by 100, from 100 to 1000.

4. Read the following:

CCC [3 hundred]   MCD [1400]   MCMLXIV [1964]

5. Tell what rule was applied to combine each of the following:

XV   CM   DIII

6. Show that:

XIX + XXXI = L [19 + 31 = 50]   M - CD = DC [1000 - 400 = 600]
C - XL = LX [100 - 40 = 60]   MDCC + CCC = MM [1700 + 300 = 2000]

7. From each set of numerals select the numeral that does not belong. Explain your reasoning.

25 + 50	75	LXXV	95 - 15	[95 - 15]
600	DC	CD	500 + 100	[CD]
400 + 100 + 6	DXXVI	526	2 x 263	[400+100+6]

8. In the numerals XLIII and LXIII the same symbols are used. Why do they not name the same number?

9. In adding CMLII and CDX, which of the following is correct? Why

CM + CD + LXII ,   MDCCLXII

* 10. What historical event do you associate with each of these Roman Numerals?

MDCCLXXVI [1776]   MCDXCII [1492]   MDCCCXLIX [1849]
MDCCCLXIII [1863]

* 11. Try to multiply XLVII by XXXVII. Then multiply 47 by 37. Compare the two products.

* 12. Why do you think the Romans did not include a symbol for zero with their numerals?

SETS; NUMBER; NUMERATION

UNIT 46 - NUMERATION: EXTENDING UNDERSTANDING OF DECIMAL (BASE 10) SYSTEM

Objectives: To develop Place Value through Billion
To extend ability to use expanded notation

TEACHING SUGGESTIONS

1. Evaluate children's understanding of the Decimal System of Numeration.
Suggested questions follow:
Draw a line under the numeral that applies to the word symbol.

Three million five thousand twenty-eight

3,005,280 3,005028 3,500,028 None of these

Draw a line under the "6" which represents six million.

6,260,000 56,351 7,627,000 326,803 None of these

How many sets of 10 objects can be combined to make a set of 2160 objects?

6 21 160 216

How many sets of 100 objects yield 7000 objects?

7 70 700 7000

How many thousands does it take to make a million?

1 thousand 10 hundred 1 hundred 10 thousand

The circled 5 is how many times as great as the underlined 5?

⑤5 ,$\underline{5}$5 5

500 10,000 1,000 100 [100]

Which of these means

10,000: 100 thousand 100 hundreds 100 tens None of these

[100 hundreds]

100,000: 1000 thousands 1000 tens 100 hundreds None of these
[none]

1,000,000: 1000 tens 1000 hundreds 1000 thousands None of these
[1,000 thousands]

2. Reinforce place value through millions.

Record the numeral 1,111,111

Have children record the same numeral on a place value chart and indicate periods.

Millions			Thousands			Units		
		Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
		1	1	1	1	1	1	1

Discuss:

The role of "ten" in our system of numeration indicating values as shown below.

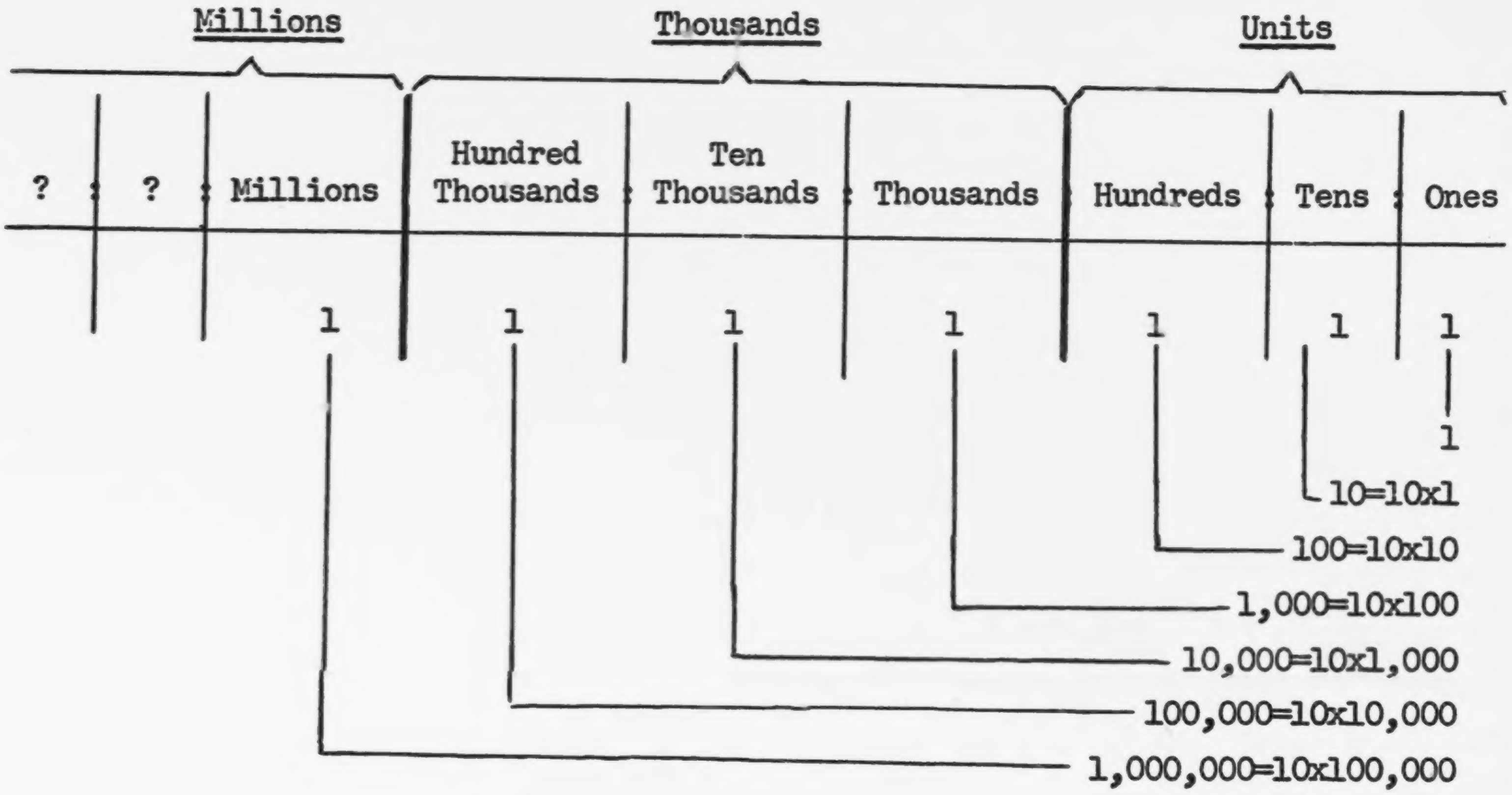
(10 sets of 10 for 100; 10 sets of 100 for 1000; etc.)

The number of digits; Why ten digits are sufficient.

The Place Value represented by each digit (ten times as great as the same digit one place to the right; $\frac{1}{10}$ as great as the same digit one place to the left)

3. Develop Place Value through billion.

A Place Value chart such as the following may be presented.


Millions
Thousands
Units
?
?
Millions
Hundred Thousands
Ten Thousands
Thousands
Hundreds
Tens
Ones
1
1
1
1
1
1
1
1
10=10x1
100=10x10
1,000=10x100
10,000=10x1,000
100,000=10x10,000
1,000,000=10x100,000

Ask children to:

Record the digit 1 to the left of millions place.
Suggest a label for the unmarked place and reasons for your choice.
Evaluate each suggestion in terms of the 1 and 10 relationship.
[10 sets of 1,000,000 are 10,000,000]
Label the unmarked place Ten Millions.
Extend the chart to show 10,000,000= 10 x 1,000,000
Extend the chart to show hundred million, to billion, etc.
For example, 10 x 10,000,000 = 100,000,000

* Discuss one billion in England vs. one billion in America.
(In England one billion = one million million)

4. Repeat development above substituting the digit 2 in each place.

Have children compare the 2 in the tens place with the 2 in the ones place.

What does the 2 in the tens place mean? [2 tens or 20]
Twenty is how many times as large as 2? [10 times]
Can you name 20 in another way? [2 sets of 10; 2 x 10]

Have children compare the 2 in the hundreds place with the 2 in the tens place and continue as with tens.

Children should continue the comparisons, moving one place at a time to the left.

5. Extend understanding of expanded notation.

Children complete the following:

$$77{,}777 = 70{,}000 + \square + 700 + \Delta + 7$$
$$= (7 \times n) + (7 \times 1{,}000) + (7 \times 100) + (7 \times 10) + (7 \times 1)$$

$$5{,}243 = \square + \Delta + 40 + 3$$
$$= (5 \times n) + (2 \times 100) + (4 \times n) + (3 \times 1)$$

EVALUATION and / or PRACTICE

SUGGESTED EXERCISES

1. Separate these numerals into periods.

6769954 30003548 230000300 30023021

2. Write the following as numerals using digits:

One hundred million, fifty thousand four
Three million, five thousand ten
Thirty million, two hundred six thousand, fifty
Forty thousand forty

3. Write each of the following in two ways to show the meaning of place value.

999,999   65,327   28,043

$$\begin{bmatrix} 20{,}000 + 8{,}000 + 40 + 3 \\ (2 \times 10{,}000) + (8 \times 1000) + (4 \times 10) + (3 \times 1) \end{bmatrix}$$

✱ 4. How many complete thousands, hundreds, tens, ones are in the following:

3,214,268

$$\begin{bmatrix} 3{,}214 \text{ thousands} \\ 32{,}142 \text{ hundreds} \\ 321{,}426 \text{ tens} \\ 3{,}214{,}268 \text{ ones} \end{bmatrix}$$

547

$$\begin{bmatrix} \text{No thousands} \\ 5 \text{ hundreds} \\ 54 \text{ tens} \\ 547 \text{ ones} \end{bmatrix}$$

OPERATIONS

UNIT 47 - SET OF WHOLE NUMBERS: ADDITION AND SUBTRACTION

Objectives: To maintain computational skill in Addition and Subtraction
To provide practice in applying Properties of Addition and Subtraction

TEACHING SUGGESTIONS

1. Provide practice in:

Applying the Associative Property of Addition for all whole numbers: $(a+b) + c = a + (b+c)$

Have children complete the equations below. Regroup the second addend "crossing a ten". Children should understand that in each case "8" is renamed as (5+3) first. Then by the associativity of addition:

$$5 + 8 = (5+\square) + 3 = n \quad [\text{here } 5 + (5+3) = (5+5) + 3]$$
$$25 + 8 = (25 + \square) + 3 = n$$
$$235 + 8 = (235 + 5) + \square = n$$
$$3225 + 8 = (3225 + \square) + 3 = n$$

Using Inverse Operations

"Adding n" and "subtracting n" are inverse operations because one undoes what the other does. For example:

$$12 + 4 = 16 \text{ vs. } 16 - 4 = 12$$

$$13 - 5 = 8 \text{ vs. } 8 + 5 = 13$$

Children relate adding to subtracting.

$$9 + 4 = 13 \qquad 19 + 4 = 23$$
$$13 - \square = 9 \qquad \square - 4 = 19$$

Then extend to adding to and to subtracting from numbers in the hundreds, thousands.

$119 + 4 = n$ $\qquad$ $6758 + 7 = n$

$\square - 4 = 119$ $\qquad$ $\square - 7 = 6758$

Applying the Commutative Property of Addition: $a + b = b + a$ for all whole numbers a, b.

Present:

Higher decade addition, e.g. $6 + 39 = 39 + \square = n$;
$7 + 48 = 48 + \square = n$; etc.

Adding to numbers in the hundreds, e.g. $3 + 138 = 138 + \square = n$
$5 + 246 = 246 + \square = n$

Adding to numbers in the thousands, e.g. $2 + 1119 = 1119 + \square = n$
$5 + 2238 = 2238 + \square = n$

Children should summarize:

> The order of the addends may be interchanged without changing the sum. Reversing the addends sometimes helps to make an addition easier: e.g.
>
> $7 + 5$ is easier than $5 + 7$
>
> $1119 + 2$ is easier than $2 + 1119$

2. Evaluate understanding of the application of mathematical principles.
Complete the following statements:
Since $748 + 76 + 393 = 1217$, $\qquad$ $76 + \square + 748 = 1217$

For $960 + 750 = n$; underline the expressions below that can replace n.

$(960 + 700) + 50$ $\qquad$ (9 + 60) +) 7 + 50)
$(900 + 700) + (60 + 50)$ $\qquad$ $(9 + 7) + (60 + 50)$

Change each false statement to make it a true statement

$76 + 0 = 0 + 76$ $\qquad$ $97 - 58 = 58 - 97$

$127 - 9 = 128 - 10$ $\qquad$ $230 + 70 = 70 + 230$

Determine an efficient method for evaluating each of the following. Explain.

2654 + 2999 + 3999 = n [2654 + 3000 + 4000 - 2]

6542 - 4998 = n [6542 - 5000 + 2]

1218 + 1219 = n [1218 + 1218 + 1]

3050 - 1526 = n [3050 - 1525 - 1]

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. If T = 625, R = 245 and V = 875, find the value of n.

   T + V = n [1500] V - T = n [250] T + R + V = n [1745]

   T + n = V [250] T - n = R [380]

2. Arrange the following in increasing order beginning with the sum or remainder of least value.

   Solve each example to check your work.

Set A	Set B
3475 - 240	13,509 + 417
3475 - 218	13,509 + 421
3475 - 256	13,509 + 468
3475 - 231	13,509 + 442
3475 - 264	13,509 + 439

3. Replace each frame with a numeral and solve for n.

   375 + 75 = 375 + □ + 50 = n 215 - 45 = 215 - □ - 30 = n

   185 + 45 = 185 + □ + 30 = n 450 - 75 = 450 - □ - 25 = n

   1265 + 55 = 1265 + □ + 20 = n 118 - 36 = 118 - □ - 18 = n

4. Write an equation to find the perimeter of a rectangle whose width is 36" and whose length is 78".

5. Write the equation to find the temperature at 10 P.M. if it dropped 14° from the 8 A.M. temperature of 23°.

* 6. Suppose $A + B + C = D$ **(Optional)**

  a. If B was doubled, how much would D increase?

  b. If each addend was doubled, how much would the sum be increased?

  c. If A was increased by 6 and C was decreased by 12, what would be the effect on D.

7. Select suitable verbal problems from textbooks and from other curriculum areas.

OPERATIONS

UNIT 48 - SET OF WHOLE NUMBERS: MULTIPLICATION; PROPERTIES APPLIED; HORIZONTAL AND VERTICAL FORMAT

Objectives: To extend multiplication of whole numbers: products in the tens of thousands; one factor through 9.

To give practice in multiplying with dollars and cents.

To introduce multiplication with both factors through 99.

To extend development involving one factor through 99, the other factor in the hundreds.

TEACHING SUGGESTIONS

1. Continue to provide practice. Refer to units 36, 37, 38 and textbooks for: equations, patterns, procedures in computation, (horizontal and vertical format).

2. Make sure that children understand:

   Renaming numbers of larger value, e.g.

   24,246 = 20,000 + 4,000 + 200 + 40 + 6

   Place value concepts

   Regrouping for exchange

Products In the Tens of Thousands: One Factor Through 9, Vertical Format

1. Present a problem situation: An airplane pilot makes 3 trips each week to a city 4,143 miles away. How far does he travel each week?

2. Children should:

   Estimate product
   Record the estimate
   Compute
   Compare the results of the computation with the estimate
   Check the work by addition and / or the distributive property

For example, for the problem above:

Estimate: $n > 12{,}000$ miles; $n > 12{,}300$ miles

Computation:

$$\begin{array}{r} 4143 \\ \underline{\times \quad 3} \\ 12{,}429 \end{array}$$

Check: (The standard algorithm for multiplication depends upon the application of the Distributive Property of Multiplication with respect to Addition).

$$\begin{aligned} 3 \times 4143 &= (3 \times 4000) + (3 \times 100) + (3 \times 43) \\ &= 12{,}000 + 300 + 129 \\ &= 12{,}300 + 129 \\ &= 12{,}429 \end{aligned}$$

Children should compare product with estimate.

3. Suggested practice exercises.

$$\begin{array}{r} 3212 \\ \underline{\times \quad 8} \end{array} \qquad \begin{array}{r} 1402 \\ \underline{\times \quad 9} \end{array} \qquad \begin{array}{r} 6058 \\ \underline{\times \quad 4} \end{array} \qquad \text{etc.}$$

Dollars and Cents

Horizontal Format

1. Suggested Multiplication Exercises: Products through \$999.99

Ask children to solve the following using the Distributive Property of Multiplication with respect to Addition:
For example:

$$\begin{aligned} 2 \times \$75.16 &= 2 \times (\$75 + .16) \\ &= \square + \$.32 = n \end{aligned}$$

$2 \times \$\,55.34 = \square + \$\,.68 = n$
$2 \times \$332.45 = \square + \$\,.90 = n$
$2 \times \$427.06 = \square + \$\,.12 = n$
$2 \times \$280.25 = \square + \$\,.50 = n$
$2 \times \$265.90 = \square + \$1.80 = n$
etc.

$2 \times \$\,84.26 = n + \$\,.52 = n$
$7 \times \$\,21.02 = n + \$\,.14 = n$
$3 \times \$323.30 = n + \$\,.90 = n$
$6 \times \$104.10 = n + \$\,.60 = n$
$4 \times \$215.40 = n + \$1.60 = n$
etc.

Vertical Format

2. Introduce exercises which involve products through \$999.99. For example:

a. $\begin{array}{r}\$86.87\\ \underline{\mathrm{x}\ \ 2}\end{array}$ Estimate: $n > \$160$ or
$n > \$172$

Compute:

$$\begin{array}{r}\$86.87\\ \underline{\mathrm{x}\ \ 2}\\ \$\underline{173}.74\end{array}$$

Children note the regrouping of tens of dollars as hundreds and tens.

Check: $\begin{array}{r}\$\ 86.87\\ \underline{86.87}\\ \$173.74\end{array}$ or $\begin{array}{rr}2 \mathrm{\ x\ } \$86. & = \$172.00\\ 2 \mathrm{\ x\ } \$\ \ .87 & = \underline{\$\ \ 1.74}\\ & \$173.74\end{array}$

b. $\begin{array}{r}\$72.14\\ \underline{\mathrm{x}\ \ 5}\end{array}$ Estimate: More than \$350 or
More than \$360

Compute:

$$\begin{array}{r}\$\ 72.14\\ \underline{\mathrm{x}\ \ 5}\\ \$360.70\end{array}$$

Children note that 36 tens of dollars is regrouped as 3 one hundred dollars and 6 tens of dollars.

Check: By addition or by applying the Distributive Property of Multiplication with respect to Addition.

c. $\begin{array}{r}\$235.61\\ \underline{\mathrm{x}\ \ 3}\end{array}$ Estimate: Between \$600 and \$715

Compute:

$$\begin{array}{r}\$235.61\\ \underline{\mathrm{x}\ \ 3}\\ \$706.83\end{array}$$

Children note the value and position of each digit of the numeral in the product.

Check: By addition or by applying the Distributive Property of Multiplication with respect to Addition.

Both Factors Through 99

Horizontal Format

1. Make sure children are able to multiply by 10 and multiples of 10.

2. Reinforce additions needed to derive unknown products from known products. For example:

Adding Groups in Sequence	Doubling Groups
10 x 34 = n (read as 10 thirty-fours)	10 x 34 = ?
20 x 34 = n + 340 = ?	20 x 34 = □ + □ = n
30 x 34 = n + 340 = ?	40 x 34 = □ + □ = n
40 x 34 = n + 340 = ?	80 x 34 = □ + □ = n
through	
90 x 34 = n + 340 = ?	30 x 34 = □
	60 x 34 = □

Doubling and Adding Groups

10 x 34 = □	30 x 34 = ?
20 x 34 = □ + □ = ?	60 x 34 = ?
30 x 34 = □ + 340 = ?	70 x 34 = □ + 340 = n
40 x 34 = □ + □ = ?	30 x 34 = 1020
80 x 34 = □ + □ = ?	60 x 34 = □ + □
90 x 34 = □ + 340 = ?	90 x 34 = □ + 1024 = n

3. Suggested development

a. For 11 x 39 = n

A child may say:	Teacher could record:
10 thirty nines = 390	10 x 39 = 390
1 thirty nine = 39	1 x 39 = 39
390 and 30 are 420 and 9 more are 429	390 + 30 + 9 = 429

b. For 21 x 34

A child may say:	Teacher could record:
10 thirty fours = 340	10 x 34 = 340
20 thirty fours = 680	20 x 34 = 680
1 thirty four = 34	1 x 34 = 34
680 and 34 = 714	680 + 34 = 714

c. Have children complete exercises like the following:

20 x 36 = n + n = n
40 x 36 = n + n = n
41 x 36 = n + 36 = ? Why?

? x 23 = 230 ? x 12 = 120
? x 20 = 400 ? x 30 = 900 etc.

23 x 16 = n + 48 = ?
21 x 83 = n + 83 = ?
27 x 32 = n + 224 = ?

d. Have children change each false sentence to make it a true sentence.

12 x 104 = 1040
14 x 216 = 2160 + 800
20 x 379 = 3790 + 3790
9 x 246 = 2460 - 10
16 x 138 = 6 x 138 plus 6 x 138
32 x 17 = 17 x 23

Vertical Format

Reinforce children's understanding that:

> The standard algorithm for multiplication depends upon the application of the Distributive Property of Multiplication with respect to Addition.

1. Present problem:

> Copies of the school newspaper are being sent to 14 classes this morning. If each class receives 32 copies, how many newspapers will be distributed?

2. Have children suggest various ways of solving the multiplication.
3. Introduce the conventional form. Suggest that children compute beginning with the ones.

$$\begin{array}{r} 32 \text{ newspapers} \\ \underline{\times 14} \end{array}$$

Estimate: More than 320;
or
More than 440

Record: $\begin{array}{r} 32 \\ \underline{\times\ 4} \\ 128 \end{array}$ $\begin{array}{r} 32 \\ \underline{\times\ 10} \\ 320 \end{array}$ $\begin{array}{r} 128 \\ \underline{+\ 320} \\ 448 \end{array}$

$$\begin{array}{r l} 32 & \text{newspapers (refer to problem and compute again)} \\ \underline{\times 14} & \\ 128 & (4 \times 32) \\ \underline{320} & (10 \times 32) \\ 448 & \text{newspapers } (14 \times 32) \end{array}$$

4. Extend development to include products in the thousands.
For example: For the problem shown below we think of 60 as 60 + 2.
Therefore $36 \times 62 = 36 \times (60 + 2) = (36 \times 60) + (36 \times 2)$

When we compute using the conventional vertical algorithm we apply the commutative property and begin with the ones. Then
$36 \times 62 = (36 \times 2) + (36 \times 60)$

$$\begin{array}{r l} 36 & \\ \underline{\times 62} & \\ 72 & (2 \times 36) \\ \underline{2160} & (60 \times 36) \\ 2232 & (62 \times 36) = (2 \times 36) + (60 \times 36) \end{array}$$

Children may check by using the Commutative Property: $36 \times 62 = 62 \times 36$

$$\begin{array}{r} 62 \\ \underline{\times 36} \end{array}$$

5. Extend development to include one factor in the hundreds.

$$\begin{array}{r l} 136 & \\ \underline{\times\ 72} & \\ 272 & (2 \times 136) \\ \underline{9520} & (70 \times 136) \\ 9792 & (72 \times 136) = (2 \times 136) + (70 \times 136) \end{array}$$

Ask children to explain how the Distributive Property is being used here.

Multiplication exercises may be presented in various ways.

$56 \times 807 = n$
Multiply 807 by 56
Multiply 56 and 807
Find the product of 56 and 807
If one factor is 56, and the other factor is 807, what is the product?

$$\begin{array}{r} 807 \\ \underline{\times 56} \end{array}$$

__Exercises__ may be __read__ in a variety of ways.

Problem	May Be Read as:
62 x 458 = n	62 four hundred fifty eights
	62 times 458
458	458 taken 62 times
x 62	458 multiplied by 62

__Numerals__ may be __read__ in various ways. Children decide on the most convenient way for a specific purpose.

7800 may be read as: seventy eight hundred or
7 thousand 8 hundred.

__Products__ may be __estimated__ in a variety of ways.

For: 132
x 27

__Children may think:__

10 x 130 = 1300
20 x 130 = 2600
30 x 130 = 3900

or

10 x 132 = 1320
20 x 132 = 2640
30 x 132 = 3960

Children should record estimate only not the thinking involved:

27 x 132 > 2600 or 27 x 132 > 2640

6. Extend development to include products in the tens of thousands.

Suggested problem: Mr. Smith donated 596 library books to each of 62 schools in our city. How many books did Mr. Smith donate?

62 x 596 = n

Children should estimate the product first. Estimates will vary according to the ability of the child.

Estimates:

Since 10 x 500 = 5000
then 60 x 500 = 6 times as much
60 x 500 = 30,000
62 x 596 > 30,000

or

Since 10 x 600 = 6000 (596 is almost 6000)
then 60 x 600 is 6 times as much
60 x 600 = 36,000
62 x 596 is about 36,000

or

Since 60 x 600 = 36,000
then 62 x 600 is 36,000 + 1200
62 x 596 is about 37,200
Children record: n is about 37,200

Children should record estimate, compute, then compare products with estimates.

EVALUATION and / or PRACTICE

SUGGESTED EXERCISES

1. 62×596

$$\begin{array}{r}596\\ \underline{\times\ 60}\end{array} \qquad \begin{array}{r}596\\ \underline{\times\ 2}\end{array} \qquad \begin{array}{r}596\\ \underline{\times\ 62}\end{array}$$

2.

$$\begin{array}{rl}596 & \\ \underline{\times\ 62} & \\ \square & = 60 \times 596\\ \underline{\Delta} & \underline{= \ 2 \times 596}\\ n & = 62 \times 596\end{array} \qquad\qquad \begin{array}{rl}596 & \\ \underline{\times\ 62} & \\ 1192 & = \square \times 596\\ \underline{35760} & \underline{= \square \times 596}\\ 36952 & = \square \times \Delta\end{array}$$

3. Present other multiplication problems. Include zeros in one factor.

$$\begin{array}{r}368\\ \underline{\times\ 79}\end{array} \qquad \begin{array}{r}607\\ \underline{\times\ 58}\end{array} \qquad \begin{array}{r}740\\ \underline{\times\ 86}\end{array} \qquad \begin{array}{r}\$3.64\\ \underline{\times\ \ \ 58}\end{array} \qquad \begin{array}{r}\$2.98\\ \underline{\times\ \ \ 95}\end{array}$$

Check by multiplying with the tens digit first.

4. In how many different ways can you find the product of each of the following:

50×12; $6 \times 4 \times 25$; 16×125; $A \times B \times C$

5. Additional practice exercises may be found in textbooks.

OPERATIONS

UNIT 49 - SET OF WHOLE NUMBERS: DIVISION; QUOTIENTS IN THOUSANDS; INTRODUCE DIVISORS GREATER THAN NINE

Objectives: To maintain skill in division of whole numbers: divisors through 9, quotients through 999.

To extend ability to divide: quotients in the thousands, divisors through 9.

To develop dividing by numbers greater than 9.

TEACHING SUGGESTIONS

Dividing By Numbers Through 9

1. **Test** children's ability to multiply numbers through 9 by:
   Ten and Multiples of Ten (10 x 8, 40 x 8, 70 x 8)
   Hundreds and Multiples of One Hundred (100 x 7, 300 x 7, 800 x 7)
   Thousand and Multiples of One Thousand (1000 x 9, 2000 x 9, 1200 x 9)

2. **Test** children's ability to estimate quotients.
   Present problems with simpler dividends to less mature children.

   Suggested problem: A gift of 2864 library books was donated to our city schools. If 9 books were sent to each school, how many schools received the books? (How many nines are in 2864?)

   Estimate: Children may think:
   $100 \times 9 = 900$
   $200 \times 9 = 1800$
   $300 \times 9 = 2700$
   $400 \times 9 = 3600$

   Children record $n > 300$, $n < 400$
   n is between 300 and 400

3. **Suggested Problem:** How many sets of sixes in a collection of 5871 things?

Estimate: $n < 1000$; $n > 900$; n is between 900 and 1000.

Computation:

```
   978
6)5871 |
  5400 | 900
   471 |
   420 |  70
    51 |
    48 |   8
     3 | 978
```

Verify the answer by using the Distributive Property of Multiplication with respect to Addition.

```
900 sixes = 5400
 70 sixes =  420
  8 sixes =   48
978         5868
```

Answer: 978 sets of six and 3 left over.

978 sixes = 5868;
5868 + 3 = 5871

Children compare solution with estimate.

4. Introduce problems with quotients in the thousands.
Children should continue to arrive at answers in a variety of ways depending upon their level of ability. For example:

```
   1405
3)4216 |
   900 |  300
  3316 |
  1800 |  600
  1516 |
  1500 |  500
    16 |
    15 |    5
     1 | 1405
```

```
   1405
3)4216 |
  1500 |  500
  2716 |
  1500 |  500
  1216 |
  1200 |  400
    16 |
    15 |    5
     1 | 1405
```

Using 1000 in the partial quotient

```
   1405
3)4216 |
  3000 | 1000
  1216 |
  1200 |  400
    16 |
    15 |    5
     1 | 1405
```

Using 1000 and multiples of 100 in the partial product

```
   2553
3)7659 |
  3000 | 1000
  4659 |
  3000 | 1000
  1659 |
  1500 |  500
   159 |
   150 |   50
     9 |
     9 |    3
       | 2553
```

Shortening the Computation

```
   2553
3)7659 |
  6000 | 2000
  1659 |
  1500 |  500
   159 |
   150 |   50
     9 |
     9 |    3
       | 2553
```

5. Present other exercises.

$7\overline{)489}$ $9\overline{)1623}$ $6\overline{)23614}$ $8504 \div 8$

How many nines are there in 28,642? $n \times 8 = 1728$

Dividing By Numbers Greater Than 9

1. Test ability to multiply numbers through 99 by tens; by hundreds. For example:

$20 \times 20 = n$ $30 \times 20 = n$ $20 \times 45 = n$
$40 \times 20 = n$ $60 \times 20 = n$ $30 \times 45 = n$
$80 \times 20 = n$ $90 \times 20 = n$ $40 \times 45 = n$

$100 \times 10 = n$ $200 \times 40 = n$ $200 \times 34 = n$
$200 \times 10 = n$ $400 \times 40 = n$ $400 \times 34 = n$
$300 \times 10 = n$ $500 \times 40 = n$ $800 \times 34 = n$

2. Emphasize estimating quotients first and as a check.

Exercise: How many twenty-threes in 851?

$23\overline{)851}$ $851 \div 23 = n$ $n \times 23 = 851$

By Doubling Numbers	By Doubling and Adding Numbers
Since $10 \times 23 = 230$ $20 \times 23 = 460$ and $40 \times 23 = 920$	Since $10 \times 23 = 230$ $20 \times 23 = 460$ and $30 \times 23 = 690$ and $40 \times 23 = 920$
Since the dividend is 851; the quotient < 40.	$30 \times 23 = 690$. Since the dividend is 851; the quotient > 30.

Children should record: Estimate: $n > 30$, $n < 40$, n is between 30 and 40.

Then check solution with estimate and verify the quotient.

3. Development

Problem: Tom's job is to distribute the school newspaper to the children on the fourth floor. He has 238 copies. If each class receives 32 newspapers, how many classes will get copies from Tom?

Children should interpret problem: (dividing the papers into stacks or groups of 32).

They can then record symbols: 32)238

Then interpret the symbols: How many thirty-twos are there in 238?

Ask children to solve the problem beginning with as many thirty-twos as they wish.

The first solution should be recorded on the chalkboard as the teacher and the children work together.

Ask the children to solve the same problem in other ways. Various solutions follow:

```
      7                  7                  7
32)238 |           32)238 |           32)238 |
    64 | 2              96 | 3            160 | 5
   174 |               142 |              78 |
   128 | 4              96 | 3             64 | 2
    46 |                46 |              14 | 7
    32 | 1              32 | 1
    14 | 7              14 | 7
```

Solution: 7 classes
14 extra newspapers

Compare the various solutions.

Have children check the quotient by:

Multiplying the quotient by the divisor and adding the remainder.

```
  32 newspapers           224 newspapers
x  7                    + 14 newspapers
 224 newspapers           238 newspapers
```

4. During the initial presentation of division exercises different methods should be encouraged and discussed.

```
     12                12                12                12
23)289 |          23)289 |          23)289 |          23)289 |
    69 | 3            115 | 5            92 | 4           230 | 10
   220 |              174 |             197 |            59 |
   138 | 6            115 | 5           184 | 8            46 | 2
    82 |               59 |              13 | 12           13 | 12
    69 | 3             46 | 2
    13 | 12            13 | 12
```

Solution: 12 groups, 13 left over

The advantage of beginning with 10 as the first partial quotient should be noted.

```
      12                  14                  17
  31)392              17)249              24)412
     310 | 10            170 | 10            240 | 10
      82 |                79 |               172 |
      62 |  2             68 |  4            120 |  5
      20 | 12             11 | 14             52 |
                                              48 |  2
                                               4 | 17
```

Children can check by using: The Distributive Property of Multiplication with respect to Addition.

```
      12           10 x 31 = 310
  31)392            2 x 31 =  62           31
     310 | 10                            x 12
      82 |          310       372   or     62         372
      62 |  2      + 62      + 20         310        + 20
      20 | 12       372       392         372         392
```

Various algorithms for 23)$\overline{719}$ follow:

```
      31                  31                  31
  23)719              23)719              23)719
     230 | 10            460 | 20            690 | 30
     489 |               259 |                29 |
     230 | 10            230 | 10             23 |  1
     259 |                29 |                 6 | 31
     230 | 10             23 |  1
      29 |                 6 | 31
      23 |  1
       6 | 31
```

Solution: 31 groups and 6 left over.

Check: Use Distributive Property

Various algorithhns, for $31\overline{)1679}$ follow:

```
      54                      54                      54
31)1679 |              31)1679 |              31)1679 |
    620 |  20              1240 |  40             1550 |  50
   1059 |                   439 |                  129 |
    620 |  20               310 |  10              124 |   4
    439 |                   129 |                    5 |  54
    310 |  10               124 |   4
    129 |                     5 |  54
    124 |   4
      5 |  54
```

5. Provide practice. Relate to problem situations.

   a. $14\overline{)\,86}$ $12\overline{)\,95}$ $21\overline{)\,87}$ $41\overline{)\,91}$ $24\overline{)\,85}$ etc.

   $23\overline{)137}$ $31\overline{)192}$ $25\overline{)217}$ $15\overline{)137}$ $24\overline{)189}$ etc.

   $21\overline{)2364}$ $41\overline{)1765}$ $52\overline{)2137}$ $33\overline{)2638}$ etc.

   b. If the perimeter of a square garden plot is 576 ft., how long is each side?

   c. If an elevator may carry at most 23 people at a time, how many trips must it make to carry down 169 people?

   d. If a grocer has egg cartons that hold a dozen eggs, how many will he need to hold 418 eggs?

6. Additional practice exercises and verbal problems may be found in textbooks.

SETS; NUMBER; NUMERATION

UNIT 50 - SET OF FRACTIONAL NUMBERS: SIXTEENTHS; LOCATING OTHER FRACTIONS ON THE NUMBER LINE

Objectives: To develop concept of sixteenths
To help children make comparisons among fractions
To help children locate $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc. of fractions on number line

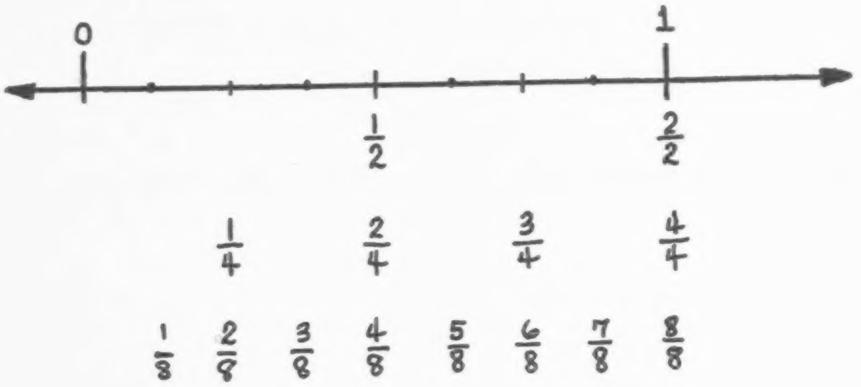
TEACHING SUGGESTIONS

Locating Sixteenths on the Number Line By Successive Bisection

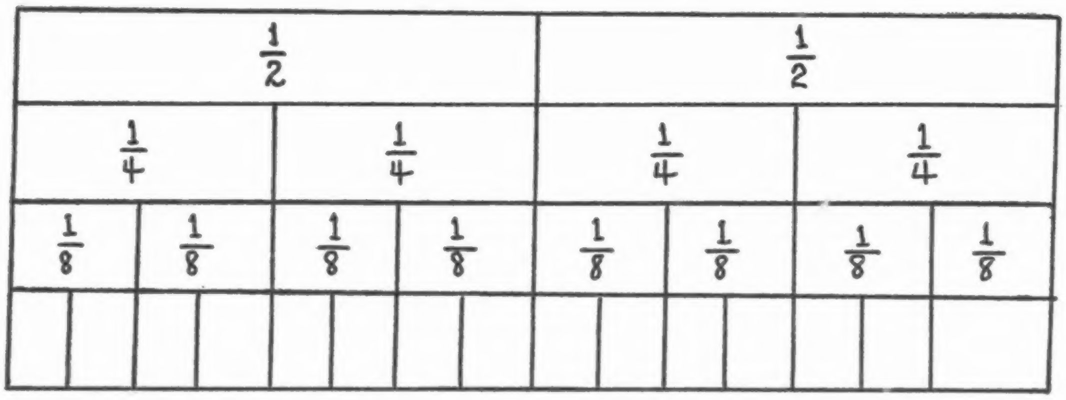
Using One-Half as an Operator

1. Reinforce finding eighths. Use a number line.

   Children should draw part of a number line showing 1 unit of length. They divide this segment into 2 equal parts and indicate the midpoint $\left(\frac{1}{2}\right)$, into 4 equal parts and label the endpoint of each part; into 8 equal parts and label each endpoint.


0
1
$\frac{1}{2}$ $\frac{2}{2}$
$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$
$\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$

2. Have children develop a chart like the one below by dividing each eighth interval into two equal parts.


$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$
$\frac{1}{8}$

Ask children:

What name can we give each unlabeled part? $\left[\frac{1}{16}\right]$ Why?

How many sixteenths are in $\frac{1}{8}$? [2] in $\frac{2}{8}$? . . . . . $\frac{8}{8}$?

How many sixteenths are in $\frac{1}{4}$? [4] in $\frac{1}{2}$? in 1 ?

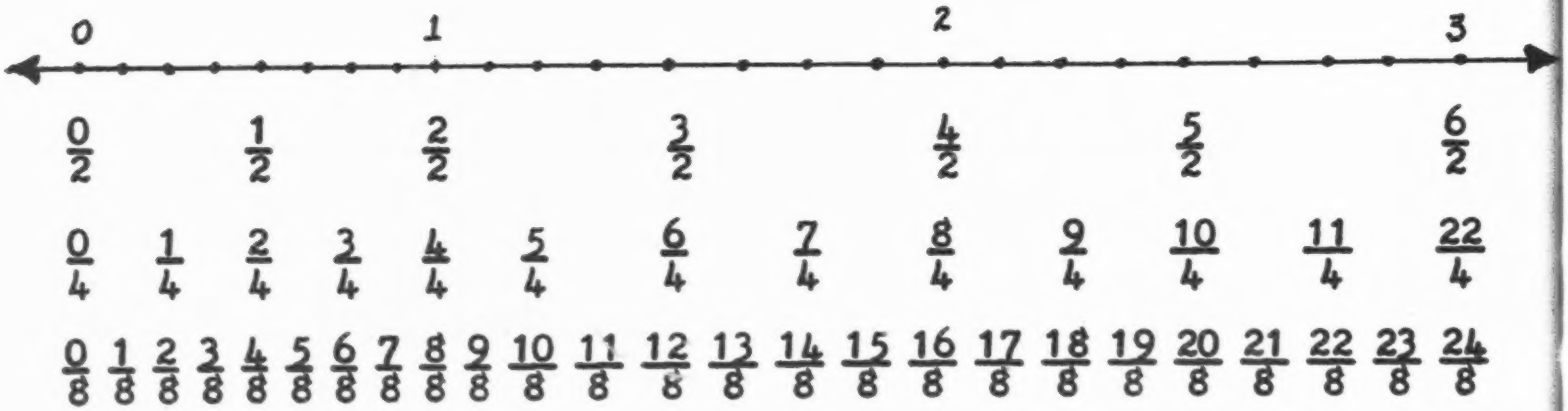
3. Have children use this chart, a number ray, and the symbols > and < to complete the sentences below.

$\frac{1}{2}$ ___ $\frac{1}{\text{[illegible]}}$ [>] $\frac{3}{8}$ ___ $\frac{1}{4}$ [>] $\frac{3}{4}$ ___ $\frac{5}{8}$ [>]

$\frac{1}{8}$ ___ $\frac{1}{4}$ [<] $\frac{1}{2}$ ___ $\frac{3}{8}$ [>] $\frac{3}{8}$ ___ $\frac{2}{4}$ [<]

$\frac{3}{16}$ ___ $\frac{3}{8}$ [<] $\frac{7}{8}$ ___ $\frac{9}{16}$ [>] $\frac{5}{16}$ ___ $\frac{3}{4}$ [<]

Children refer to the number line below to help answer the questions suggested.


0
1
2
3
0/2 1/2 2/2 3/2 4/2 5/2 6/2
0/4 1/4 2/4 3/4 4/4 5/4 6/4 7/4 8/4 9/4 10/4 11/4 22/4
0/8 1/8 2/8 3/8 4/8 5/8 6/8 7/8 8/8 9/8 10/8 11/8 12/8 13/8 14/8 15/8 16/8 17/8 18/8 19/8 20/8 21/8 22/8 23/8 24/8

Which number is smaller, $\frac{17}{8}$ or $\frac{16}{8}$? $\left[\frac{16}{8}\right]$

Which is farther to the left on the number line, $\frac{16}{8}$ or $\frac{17}{8}$? $\left[\frac{16}{8}\right]$

Which number is smaller $\frac{10}{16}$ or $\frac{12}{8}$? $\left[\frac{10}{16}\right]$

Which is farther to the left on the number line? $\left[\frac{10}{16}\right]$

Which number is smaller, $\frac{17}{8}$ or $\frac{15}{8}$? $\left[\frac{15}{8}\right]$

Which is farther to the left on the number line? $\left[\frac{15}{8}\right]$

Which number is smaller $\frac{16}{16}$ or $\frac{4}{2}$? $\left[\frac{16}{16}\right]$

Which is farther to the left on the number line ?

$\frac{9}{16}$ **or** $\frac{1}{2}$ **? ;** $\frac{0}{8}$ **or** $\frac{0}{16}$ **?**

Relate to music: **a quarter note is given half the time of a half note; an eighth note is given half the time of a quarter note; etc.**

4. Compare a number line with a ruler graduated in sixteenths. Have children show with rulers why $\frac{1}{8}$ may be interpreted as

$\frac{2}{16}$; $\frac{1}{4}$ as $\frac{4}{16}$; etc.

Locating Other Fractions Using One-Half As The Operator

1. Ask children to draw another number line. Divide a unit length into 2 equal parts. Label each part.

   If each sixteenth were divided into 2 equal parts, how many equal parts would there be in 1 interval of length? [32]

   What is each part called? $\left[\frac{1}{32}\right]$

   Children note that taking $\frac{1}{2}$ of a number creates a new number one half the size. Discuss further subdivisions into 2 equal parts (sixty-fourths, one hundred twenty-eighths, etc.)

   What happens to the value of the fraction as the denominator increases? decreases? is doubled? is halved? Why?

2. Have children continue to use number lines and rulers where possible to locate other fractions.

3. Discuss :

   a. The meaning of:

   $\frac{1}{6}$ as $\frac{1}{2}$ of $\frac{1}{3}$ $\quad$ $\frac{1}{12}$ as $\frac{1}{2}$ of $\frac{1}{6}$ $\quad$ $\frac{1}{24}$ as $\frac{1}{2}$ of $\frac{1}{12}$

   $\frac{1}{14}$ as $\frac{1}{2}$ of $\frac{1}{7}$ $\quad$ $\frac{1}{28}$ as $\frac{1}{2}$ of $\frac{1}{14}$ $\quad$ $\frac{1}{56}$ as $\frac{1}{2}$ of $\frac{1}{28}$ etc.

   b. The coordinate point halfway between:

   0 and 1 $\quad$ 0 and $\frac{1}{2}$ $\quad$ 0 and $\frac{1}{4}$ $\quad$ 0 and $\frac{1}{16}$

   etc.

   1 and $1\frac{1}{8}$ $\left[\frac{1}{16}\right]$ $\quad$ 1 and $1\frac{1}{16}$ $\left[\frac{1}{32}\right]$ $\quad$ 1 and $\frac{1}{32}$ $\left[\frac{1}{64}\right]$

Can we continue to divide any segment into 2 equal parts?
What is the limit of such subdivisions?

Locating Other Fractions Using One-Third, One-Fourth, etc. As Operators

Suggested exercises: Use number line or other aids when desirable.

1. $\frac{1}{3}$ **of** $\frac{1}{2} = \frac{1}{\square}$ $\quad$ $\frac{1}{3}$ **of** $\frac{1}{5} = \frac{1}{\square}$ $\quad$ $\frac{1}{3}$ **of** $\frac{1}{10} = \frac{1}{\square}$

What part of $\frac{1}{6}$ **is** $\frac{1}{18}$**?** $\quad$ $\frac{1}{6}$ **is how many times** $\frac{1}{18}$**?**

2. **Show different ways of obtaining** $\frac{1}{16}$, $\frac{1}{18}$, $\frac{7}{24}$, **etc. as the result of subdividing a unit interval.**

3. **Complete the following:**

Since $\frac{1}{2}$ **of** $\frac{1}{12} = \frac{1}{24}$ $\quad$ **Since** $\frac{1}{3}$ **of** $\frac{1}{5} = \frac{1}{15}$
then $\frac{1}{2}$ **of** $\frac{2}{12} = \frac{\square}{24}$ $\quad$ **then** $\frac{1}{3}$ **of** $\frac{2}{5} = \frac{\square}{15}$
and $\frac{1}{2}$ **of** $\frac{5}{12} = \frac{\square}{24}$ $\quad$ **and** $\frac{1}{3}$ **of** $\frac{4}{5} = \frac{\square}{15}$

Since $\frac{1}{4}$ **of** $\frac{1}{10} = \frac{1}{40}$
then $\frac{1}{4}$ **of** $\frac{2}{10} = \frac{\square}{40}$
and $\frac{1}{4}$ **of** $\frac{7}{10} = \frac{\square}{40}$

EVALUATION and / or PRACTICE SUGGESTED EXERCISES

1. **Tell whether we are considering a number or a numeral when we:**

Add fractions	**[number]**
Use the decimal form	**[numeral]**
Change fractions to simpler form	**[numeral]**
Compare fractions	**[number]**

2. If the interval on a ruler between zero and one is divided into 8 equal parts, what is each equal part called?

3. Draw part of a number line. Label the points that correspond to the numbers 0, 1, 2, 3. Consider the distance between 0 and 1 as the unit of length. Divide each unit of length into 3 equal parts. Label these points.

4. Label the points on the number line that represent the numbers:

$$1\frac{2}{3}, \quad 2\frac{1}{3}, \quad \frac{5}{6}, \quad \frac{7}{6}, \quad \frac{5}{12}$$

5. List 3 fractional numerals that name 0; 1; 2; etc.

6. Rename $\frac{6}{2}$ in 4 different ways.

7. Compare $\frac{3}{5}$ with $\frac{5}{3}$. How do you know which of them is less than 1? more than 1?

8. In the fraction $\frac{12}{12}$, give an interpretation to 12 in the numerator, the 12 in the denominator.

9. Using a ruler with one-fourth inch markings, how can we measure $1\frac{3}{8}$ inches?

10. There is a marker on the Thruway for every tenth of a mile. How many markers are there in $\frac{3}{5}$ of a mile? in $1\frac{3}{10}$ miles? in 100 miles? in n miles?

11. Which of the following are names for whole numbers?

$$\frac{10}{5}, \quad \frac{7}{8}, \quad \frac{0}{3}, \quad \frac{13}{12}, \quad \frac{12}{4}, \quad \frac{6}{1}, \quad \frac{8}{9}$$

12. Insert the correct symbols >, =, < in the placeholder to compare the following:

$\frac{2}{3}$ of a yd. ☐ $\frac{2}{5}$ of a yd. $\frac{4}{12}$ of an hr. ☐ $\frac{2}{6}$ of an hr.

$\frac{1}{8}$ of a gal. ☐ $\frac{1}{2}$ of a qt.

13. Rewrite the following set in order starting with the smallest.

$\frac{9}{8}, \frac{3}{8}, \frac{7}{8}, \frac{5}{8}, \frac{11}{8}$ $\frac{3}{12}, \frac{3}{4}, \frac{3}{5}, \frac{3}{8}, \frac{3}{2}$

$\frac{3}{9}, \frac{1}{2}, \frac{3}{12}, \frac{2}{10}, \frac{1}{6}$

14. Mark each of the following statements as either True or False. Draw a number line for each pair to show the correct relationship.

$\frac{1}{2} > \frac{2}{3}$ $\frac{2}{3} < \frac{3}{4}$ $\frac{17}{8} < \frac{2}{1}$ $\frac{0}{12} < \frac{0}{6}$

15. What is the smallest number of parts into which a unit can be divided? The largest?

16. Show on the number line whether one-fourth or one-half is closer to one-third.

17. As the number of equal parts into which a unit is divided is increased, how does the size of the part change?

GEOMETRY AND MEASUREMENT

UNIT 51 - MEASUREMENT: LENGTH; SCALE DRAWING

Objectives: To extend concepts of length to fractional parts of an inch.
To emphasize approximate measurements.
To help children interpret drawings made to scale.

TEACHING SUGGESTIONS

Fractional Parts of an Inch

1. Have children estimate inches, inch, and half inch using handspan and fingers to show approximate size.

2. Provide each child with a rectangle (paper which can be easily be folded - 1" by 1 $\frac{1}{2}$ ").

   Fold the paper inch into halves. Discuss halves and compare with 1/2 inch markings on the standard ruler. Provide practice in estimating 1/2 inch (margins, space between lines, space between letters when printing, etc.) Always verify estimates.

3. Provide each child with a strip of durable paper (oaktag), 12 inches by 1 inch, for the construction of his own ruler.

   Reinforce concept of inch by having children use their inch rectangle to mark off the inches on their own oaktag ruler.

   Verify with a standard ruler.

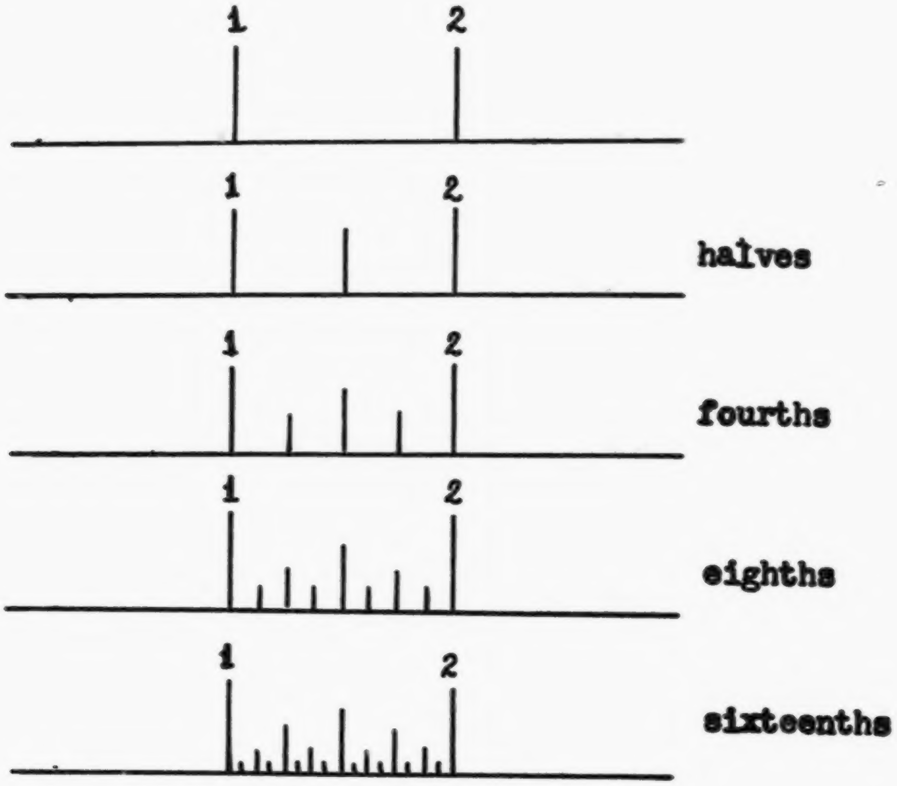
   Discuss with children a convenient place to write the numerals 1 - - - 12 on the oaktag rulers. Compare with standard rulers.

   Have children mark off lengths of $\frac{1}{2}$ inch on their rulers by using the inch rectangle folded in half.

In a similar way have children complete rulers. They can mark off fourths and eighths by relating fourths to halves (1/2 of 1/2), and eighths to fourths (1/2 of 1/4).

4. Reinforce equivalents for halves, fourths, eighths and sixteenths.

Suggestion:



5. Develop the ability to measure the following lengths: 1 inch, 2 inches, 3 inches, etc. starting with any inch marking on the ruler.

Develop the ability to measure the following lengths: 1, 2, or 3 inches starting from any half-inch marking on the ruler; from any marking.

Develop the ability to measure the following lengths: half-inch segments from inch markings, 1/4 inch segments from 1 inch markings.

6. Provide practice in:

Locating various lengths on a ruler.
Drawing various lengths, e.g., 4 inches, $2\frac{1}{4}$ inches, $3\frac{1}{2}$ inches, $5\frac{7}{8}$ inches, etc. using a ruler.

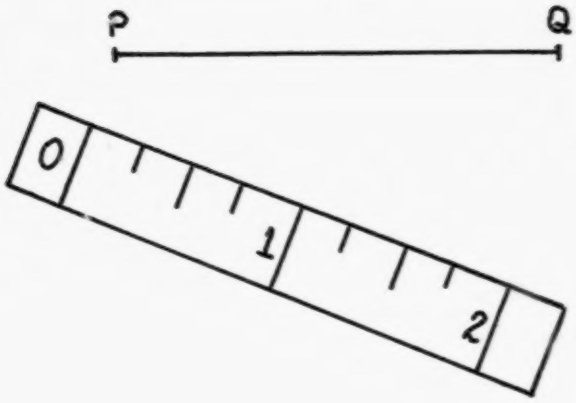
Estimating various lengths, then checking estimates. Counting using fractional parts of the inch, e.g., begin at $3\frac{7}{8}$ inches, count forward by $\frac{3}{4}$ inches; begin at $\frac{3}{16}$ of an inch count forward by $\frac{1}{2}$ inch; etc.

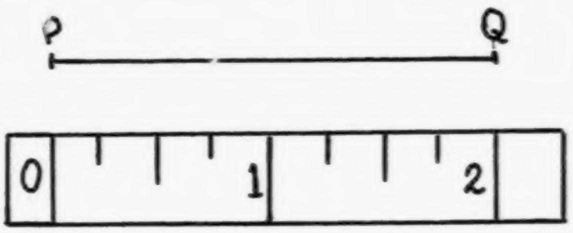
7. Ask children:

How many one-fourths of an inch are there in 1 inch: in 3 inches; in $3\frac{1}{4}$ inches?

Draw a line segment $\frac{3}{4}$ of an inch in length. How many $\frac{1}{8}$ inches in this line segment? How many $\frac{1}{4}$ inches?

Which of these drawings shows the correct way to place a ruler to measure a segment? Why?



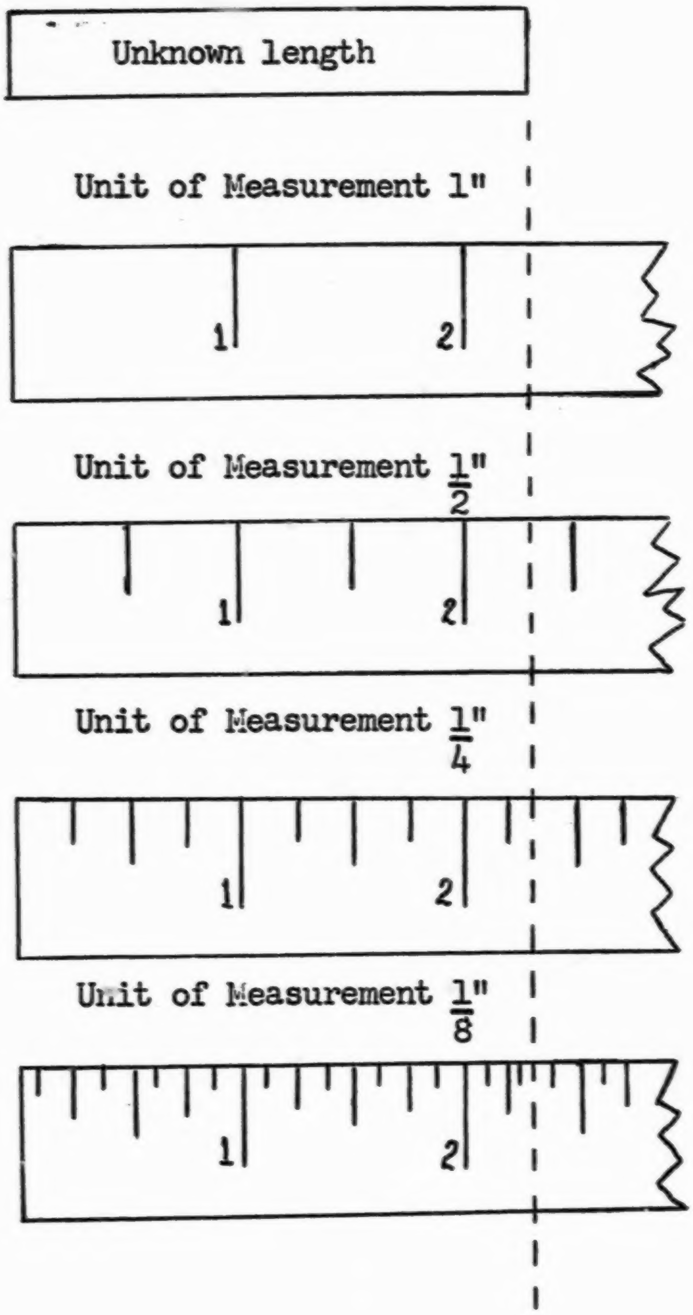


Approximate Measurements

Have children:

Examine rulers scaled in various ways: $\frac{1}{4}$ in., $\frac{1}{8}$ in., $\frac{1}{16}$ in.

Measure an unknown length to the nearest 1", $\frac{1}{2}$", $\frac{1}{4}$", $\frac{1}{8}$" as shown:



Unit of Measurement 1"

Measurement is 2" to nearest 1"

Unit of Measurement $\frac{1}{2}$"

Measurement is $\frac{5}{2}$" (2 $\frac{1}{2}$") to nearest $\frac{1}{2}$"

Unit of Measurement $\frac{1}{4}$"

Measurement is $\frac{9}{4}$" (2 $\frac{1}{4}$") to nearest $\frac{1}{4}$"

Unit of Measurement $\frac{1}{8}$"

Measurement is $\frac{19}{8}$" (2 $\frac{3}{8}$") to nearest $\frac{1}{8}$"

Have children:

Note that the numbers expressing the measurement are exact whereas the measurement is approximate.

Discuss which ruler to select to give the more precise measurement.

Note that the smaller the unit of measure used, the more precise the measurement will be to the desired length.

Measure various items, e.g., books, papers, length and width of desk, etc. with rulers of different scales.

Draw a line segment $2\frac{3}{16}$ inches using rulers of different scales.

PRACTICE EXERCISES

1. Complete each of the following and verify with a ruler.

   $\frac{1}{2}$ of $\frac{1}{4}$" = ... "

   $\frac{1}{4}$" + $\frac{1}{4}$" + ..." = $\frac{5}{8}$"

   $\frac{1}{8}$" + $\frac{1}{8}$" + $\frac{1}{8}$" + ..." = $\frac{1}{2}$"

2. Which is nearer to the 1" mark on your ruler? $\frac{5}{8}$" or $\frac{3}{4}$" ?

3. If you want to draw a segment $3\frac{3}{8}$ in. long and start at the 2" mark on your ruler, where would you stop?

Interpretation of Drawings Made to Scale

1. Examine a map of New York State.

   Discuss the mileage from New York City to Albany (about 150 miles) with the length shown on the map.

Interpret the meaning of the legend shown on the map.

Note that:

A map is a drawing made to scale.
A scale drawing is a map or diagram of a place or object that is too large or too small to draw in actual size.
A scale drawing has the same shape as the real thing it represents.

2. Present a list of items:

a baseball field	a flag	classroom
a snowflake	the school garden	Map of Mexico

Discuss for each of the above:

Should the scale drawing be larger or smaller than the actual object? Why?

What must be considered before making the drawing?

3. Draw a line segment one inch long on the chalkboard.

Tell children:

If this line segment 1 inch long, represents 1 mile, we say that "the segment is drawn to a scale of 1 inch to 1 mile".

We write: "Scale: 1 inch represents 1 mile"
or
1" = 1 mi.

What does the equal sign mean in this case?

4. Have children write two statements for each of the following:

A line segment is drawn to a scale of 1 inch to 1 foot.
A segment is drawn to a scale of $\frac{1}{2}$ in. to 10 feet.

5. Have children draw a line segment to represent $2\frac{1}{2}$ miles if:

a. 1 inch represents 1 mile

b. 1 inch represents $\frac{1}{2}$ mile

c. 1 inch represents 2 miles

SETS; NUMBER; NUMERATION

UNIT 52 - NUMBER SYSTEM: "CLOCK" ARITHMETIC

NOTE TO TEACHER

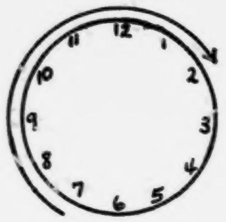
In "Clock" Arithmetic we shall be concerned with a number system that involves only a finite set of numbers, whereas in our usual arithmetic we have been dealing with an infinite set of numbers. In "Clock" Arithmetic, using a 12-hour clock, the finite set of numbers is:

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

"Clock" Addition

Let us define "addition of two clock numbers" as follows:

"3 + 2" shall mean
2 hours after 3 o'clock
Therefore, 7 + 7 = 2;
6 + 6 = 12; etc.



"Clock" addition is a binary operation in which we operate on two numbers, a and b, to arrive at a third number, c.

Some Properties of "Clock" Addition

In studying "Clock" Arithmetic, children have an excellent opportunity to discuss and discover the properties that apply to addition and subtraction of the numbers of a finite set.

Zero It should be observed that, in "Clock" Arithmetic, using a 12-hour clock, the number 12 plays the same role that zero does in our usual arithmetic. 12 is the Identity Element in "Clock" addition:
$3 + 12 = 3$; $8 + 12 = 8$; $a + 12 = a$ for all a

Closure Since the sum of any two numbers in this finite set is an element in the same set, addition is said to be closed for "Clock" Arithmetic where any number a, plus any number b, results in a number within that finite set.

Commutative Property The Commutative Property holds true in "Clock" Arithmetic. For example: $2 + 7 = 7 + 2$ (Seven hours after 2 o'clock is 9 o'clock as is 2 hours after 7 o'clock.)

Associative Property for Addition The Associative Property holds true in "Clock" Arithmetic. For example:

$$(2 + 5) + 3 = 2 + (5 + 3)$$
$$7 + 3 = 2 + 8$$

5 hours after 2 o'clock is 7 o'clock and 3 hours after 7 o'clock brings us to 10 o'clock which is the same as 8 hours after 2 o'clock.

Clock Subtraction Subtraction for "Clock" Arithmetic is defined as follows:

3 - 2 shall mean 2 hours before or earlier than 3 o'clock

Therefore, $7 - 7 = 12$;
$6 - 8 = 10$; etc.

Properties as They Apply To Subtraction

Commutativity for Subtraction does not apply. For example:

$$5 - 2 \neq 2 - 5$$

Two hours before 5 o'clock (3 o'clock) is not the same as 5 hours before 2 o'clock (9 o'clock). It is important that children understand that a property must apply in all cases in order that it be considered a property. There are special instances in subtraction with "Clock" Arithmetic where the Commutative Property does apply.

For example, using the 12-hour clock observe that:

$10 - 4 = 4 - 10$
$9 - 3 = 3 - 9$
$8 - 2 = 2 - 8$; etc.

(Note that the result each time is 6.)
However, since the Commutative Property does not apply in every case, it is not a property of Subtraction in "Clock" Arithmetic.

Associativity for Subtraction does not apply. For example:

$(8 - 3) - 2 \neq 8 - (3 - 2)$

3 o'clock is not the same as 7 o'clock

Closure for Subtraction
This finite set of numbers $\{1, 2, 3, 4 \ldots 12\}$ is closed with respect to subtraction since the difference of any two numbers is an element of this set. For example:

$8 - 3 = 5$; $4 - 6 = 10$

Note that in this respect "Clock" Arithmetic differs from our ordinary Arithmetic in which the set of whole numbers is not closed for subtraction.

OBJECTIVES:

To introduce or reinforce "Clock" Arithmetic.
To observe Properties of Addition and Subtraction in "Clock" Arithmetic.
To observe that an arithmetic based on the number of the days of the week (7) is also a system based on a finite set of numbers.
To give children an opportunity to explore, to discover.

TEACHING SUGGESTIONS

Addition and Subtraction: Using a 12-Hour Clock

1. Reinforce Addition and Subtraction in "Clock" Arithmetic using a 12-hour clock.
   Present the set of numbers: $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
   Represent this set on a clock face.
   Reinforce meaning of clockwise, counter clockwise.
   Children should explore:
      Meanings for $3 + 2$ on the clock face (2 hours after 3 o'clock)
      $5 + 7$ (7 hours after 5 o'clock), etc.
      Meanings for $12 - 3$ on the clock face (3 hours before 12)
      $3 - 4$ (4 hours before 3 o'clock)
      $6 - 12$ (12 hours before 6 o'clock); etc.

Ask children:

What is the meaning of $7 + 7 = 2$? $6 + 8 = 2$?
$8 + 5 = 1$? $3 - 9 = 6$?

Emphasize that we are dealing with only the numbers represented on the clock face and we call this "Clock" Arithmetic.

2. Suggested practice exercises:
Children should use a clock face if necessary.

a) If the time is now 4 o'clock, 6 hours later it will be __ o'clock.
If the time is now 7 o'clock 12 hours later it will be __ o'clock.
If the time is now 3 o'clock 10 hours later it will be __ o'clock.

b) In "Clock" Arithmetic
$4 + 6 = \square$ $8 + 9 = \square$
$7 + 7 = \square$ $3 + 10 = \square$

c) If the time is now 4 o'clock, 6 hours earlier it was __ o'clock.
If the time is now 7 o'clock 12 hours earlier it was __ o'clock.
If the time is now 3 o'clock 10 hours earlier it was __ o'clock.

d) In "Clock" Aritnmetic
$8 - 6 = \square$ $8 - 10 = \square$
$8 - 8 = \square$ $8 - 12 = \square$

Properties of Addition and Subtraction in "Clock" Arithmetic

1. Commutativity in Addition
Ask children to use a clock face to decide whether or not addition in "Clock" Arithmetic is Commutative. Have them give 3 examples to "prove" their decision.

2. Commutativity in Subtraction
Children should discover whether the Commutative Property holds for subtraction.
Have them use a clock face to discover whether:

$8 - 6 \overset{?}{=} 6 - 8$; (Read as: Does $8 - 6 = 6 - 8$?)
$9 - 2 \overset{?}{=} 2 - 9$;
$12 - 3 \overset{?}{=} 3 - 12$; etc.

The children discover that $8 - 6 \neq 6 - 8$, etc.

Ask children:

Do you think that the Commutative Property applies to Subtraction in "Clock" Arithmetic? [No]
Let us verify our conclusion.

Does $10 - 4 \overset{?}{=} 4 - 10$ [Yes, each equals 6]
$9 - 3 \overset{?}{=} 3 - 9$ [Yes]
$8 - 2 \overset{?}{=} 2 - 8$ [Yes]
$7 - 1 \overset{?}{=} 1 - 7$ [Yes]

Tell children that a property in mathematics can only be called a property if it applies in every case. Since the Commutative Property does not apply in every case, their first conclusion was correct.
The Property of Commutation for Subtraction does not apply to the finite set of numbers on a 12 hour clock.
They compare Commutativity for Addition and Subtraction with the System of Whole Numbers and the System of Numbers in "Clock" Arithmetic.

3. Associativity in Addition
Ask children to use the clock face to discover whether the following are true or false:

(3 + 4) + 6 = 3 + (4 + 6)
(5 + 3) + 2 = 5 + (3 + 2)

Children note that the Associative Property of Addition applies to "Clock" Arithmetic.

4. Associativity in Subtraction
In the same way have children discover that the Associative Property does not hold true for Subtraction.

5. Identity Element for Addition
Ask children to complete the following:

6 + 12 = □ 7 + 12 = □
8 + 12 = □ n + 12 = □

In "Clock" Arithmetic, which number is the Identity Element? Explain.
In the System of Whole Numbers, which number is the Identity Element?

6. Closure
Help children to discover that in "Clock" Arithmetic the set of numbers is closed with respect to Addition and Subtraction.

8 + 2 = □ 9 - 3 = □
7 = 6 = □ 6 - 8 = □
4 = 9 = □ 3 - 5 = □

Ask children:

Is the sum or difference always in the given set of numbers?
Compare this with Addition and Subtraction of Whole Numbers.

Addition and Subtraction: "Days of the Week" Arithmetic

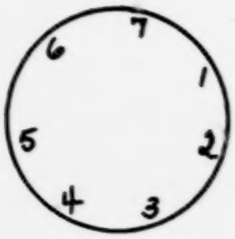
1. Discuss the numerical positions of the days of the week, referring to the calendar. Let Sunday be day 1, etc.

2. Children should refer to one week on the calendar and answer the following:

What is 3 days after Tuesday? [Friday]
What number have we assigned to Friday? [6]

Write a mathematical sentence to describe
that 3 days after Tuesday is Friday [$3 + 3 = 6$]
What is 4 days after Friday? [Tuesday]
Write a mathematical statement to show this [$6 + 4 = 3$]
Write mathematical statements to show what is
4 days after Saturday [$7 + 4 = 4$]
7 days after Sunday [$1 + 7 = 1$]

3. Have children represent the numbers for the days of the week in the same way that the numbers on the clock are represented.



Discuss similarities and differences between "Clock" Arithmetic and "Days of the Week" Arithmetic:

Both have a limited number of symbols.
In one case the set of symbols is {1, 2, 3 ... 12}.
In the other case the set of symbols is {1, 2, 3 ... 7}.
Place Value is not involved.
Zero is not used in either one.

4. Have children explore the following Properties for Addition using {1, 2, 3, 4, 5, 6, 7}.

   a. Is addition commutative? For example: $7 + 3 \overset{?}{=} 3 + 7$, etc.

   b. Is addition associative? For example: $(7 + 3) + 2 \overset{?}{=} 7 + (3 + 2)$

   c. What number of the set corresponds to the role of zero? [7]

   d. Is this finite set closed for addition?

5. Have children explore Subtraction using {1, 2, 3, 4, 5, 7}.

   a. How would you describe subtraction?

   b. Is subtraction commutative? For example:

      $7 - 3 \overset{?}{=} 3 - 7$, etc.

   c. Does the Associative Property apply to subtraction? For example:

      $(7 - 3) - 2 \overset{?}{=} 7 - (3 - 2)$

   d. Is this finite system closed for subtraction?

6. Have children compare the properties of operation when using "Clock" Arithmetic and when using "Days of the Week" Arithmetic.

SETS; NUMBER; NUMERATION

*UNIT 53 - NUMBER: THE MODULO 12 NUMBER SYSTEM (Optional)

NOTE TO TEACHER

In Clock Arithmetic we considered the following finite set of numbers:

$$C = \{1, 2, 3, .., 10, 11, 12\}$$

In Unit 52 the children were guided to discover that 12 plays the role of a zero, in that
$2 + 12 = 2$, $6 + 12 = 6$, $\square + 12 = \square$
for any replacement for Set C.
If we therefore use 0, instead of 12 we have the set:

$$\{0, 1, 2, .., 11\}$$

which we can call a "Modular Number System".

Objectives: To introduce concept of a Modular Number System.

To introduce addition in a Modulo 12 System.

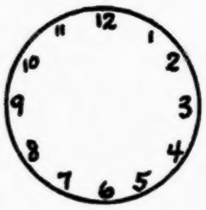
To observe the properties of addition in the Modulo 12 System.

TEACHING SUGGESTIONS

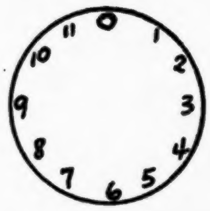
Concept of A Modular Number System

1. Teacher may use two figures as shown:

A



B



Ask children what system Figure A represents.
[A system using a 12 hour clock.]

Ask children referring to Figures A and B:

How many symbols are used in Figure A?
List that set of symbols. [{1, 2, 3,... 12}]
How many symbols are used in Figure B?
List that set of symbols. [{0, 1, 2, 3,... 11}]
Compare the 2 sets of symbols. [same number of, but different symbols]
What number in Figure B takes the place of 12 in Figure A? [0]

2. Have children answer the following:

For "Clock Arithmetic" as represented by Figure A:

12 + 2 = ☐ 12 + 6 = ☐ 12 + 9 = ☐

For Figure B:

0 + 2 = ☐ 0 + 11 = ☐ 0 + 7 = ☐

Children should note that 12 and 0 play the same role for both sets of numbers. (Identity Element)

Tell children that:

As we used the set of symbols including only {1, 2, 3,... 12} to add and subtract in "Clock" Arithmetic we are now going to use the set of symbols including only {0, 1, 2, 3,..., 11}.

We call a number system consisting of a limited number of symbols a Modular Number System.

Addition in Modulo 12 Number System

1. Develop with children a table such as the one below for adding two numbers in the Modulo 12 Number System.

+	0	1	2	3	4	5	6	7	8	9	10	11
0	[0]	[1]										
1	[1]											
2												
3												
4												
•												
•												
•												
11												

a. Teacher should reinforce concepts of a finite number system.
Ask:

What symbols are used in clock arithmetic?

[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]

How many symbols are used in clock arithmetic?
What symbols are used in the table above?
How many symbols are used in the table above?
What are the 12 symbols?

b. Have children complete the following and then insert the correct numeral on the chart.

0 + 0 = □ 0 + 1 = □ 0 + 2 = □ etc.

1 + 0 = □ 1 + 1 = □ 1 + 11 = □ etc.

c. Children complete the table.

2. Have children observe that:

In this table only a specific number of symbols are used.
No new numbers are involved. (Closure)
Place value does not apply.

Properties As They Apply to Addition in the Modulo 12 Number System.

1. Review meaning of "Addition - Modulo 12".

For example: 3 + 9 = 0; 11 + 4 = 3; etc.

2. Commutative Property

Ask children:

What is the sum of 9 + 4 as shown in this table? [1]
What is the sum of 4 + 9 as shown in this table? [1]
Is addition commutative when we use this system? [Yes] Why?

Have children give other examples to illustrate that Modulo 12 addition is commutative.

3. Associative Property

Ask children:

What is the sum of (4 + 5) + 6? [3]
What is the sum of 4 + (5 + 6)? [3]
Does (4 + 5) + 6 = 4 + (5 + 6)? [Yes]
Is this addition associative? [Yes, the way the numbers are grouped does not change the sum.]

Have children verify this with (3 + 5) + 2; (7 + 4) + 3; etc.

4. Closure

Continue to use the table to help children discover that the sum of any numbers in this finite system (Modulo 12) is always a number of this finite set.

Present addition exercises. Have children find the sums.

6 + 5 = □   10 + 4 = □   8 + 4 = □ , etc.

Note that the sum is always one of the twelve numbers in the system.

Point out that this set of numbers is closed with respect to addition.

5. Compare the Identity Element for Addition in Clock Arithmetic and the Identity Element for Addition in Modulo 12 Arithmetic.

a. Present a series of exercises referring to the clock.

1 + 12 = □
2 + 12 = □
3 + 12 = □
4 + 12 = □

9 + 12 = □
10 + 12 = □
11 + 12 = □
12 + 12 = □

12 + 5 = □
12 + 6 = □
12 + 7 = □
12 + 8 = □

Have children note that 12 is the identity element in clock arithmetic. Any number added to 12, or 12 added to any number results in that number.

Ask children:

What number in the System of Whole Numbers is the Identity Element for Addition? [0]

What number in Clock Arithmetic is the Identity Element for Addition? [12]

b. Refer to the table for Modulo 12.
Present a series of exercises.

1 + 0 = ☐ ; 3 + 0 = ☐ ; 9 + 0 = ☐ ; 0 + 11 = ☐ ; etc.

Have children note that 0 is the Identity Element for Modulo 12. Any number added to 0, or 0 added to any number results in that number.

*Other Modular Systems

NOTE TO TEACHER

Clock Arithmetic involves 12 numbers because of the way clocks are constructed. We use this as a motivation for Clock Arithmetic and Modulo 12 Arithmetic to point up arithmetic dealing with a finite number system.

Other finite number systems are important too. For example, in Grade 6, the Arithmetic Modulo 2 will take advantage of properties of even and odd numbers.

If time and interest permit, another finite number system that can be introduced for individual exploration by students is Modulo 7 Arithmetic which we can call "Days of the Week" Arithmetic making use of the periodicity of the 7 days of the week.

TEACHING SUGGESTIONS

1. Tell children that we can develop a finite number system with any number of elements greater than 1.

2. Have children explore a finite number system with 7 elements.

   They add in modulo 7.
   They find out whether properties apply to this modulo.
   e.g. For Modulo 7:

   $4 + 5 = \square$, $5 + 4 = \square$ (Commutativity)

   $(4 + 5) + 3 = \square$; $4 + (5 + 3) = \square$ (Associativity)

   $5 + 0 = \square$; $3 + 0 = \square$; $6 + 0 = \square$ (Identity)

   What is the identity element in Modulo 7?

   Children should note that the properties that apply in the set of whole numbers also apply to addition in the finite number systems.

EVALUATION AND / OR PRACTICE

SUGGESTED EXERCISES

1. Solve the following exercises and identify the property.

   $3 + 0 = 0 + \square$ (modulo 12)

   $3 + 9 = \square + 3$ (modulo 12)

   $(7 + \square) + 3 = 7 + (5 + 3)$

*2. Have children explore the Properties of Addition for Modulo 7.

*3. Make an Addition Table for Modulo 7.

*4. The Table for exercise 3 is

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3*	4	5	6	0
2	2	3*	4	5*	6	0	1
3	3	4	5*	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

Have children draw a line from the upper left hand corner to the lower right hand corner.

Children should observe that the section of the table above the line is symmetric to the section of the Table below the line.

Have children show how the commutative property is demonstrated on this grid.

Have children star 2 + 1, then 1 + 2
3 + 2, then 2 + 3 etc.

OPERATIONS

UNIT 54 - SET OF WHOLE NUMBERS: EVEN AND ODD NUMBERS

Objectives: To determine divisibility of a number by two.

To observe the results of adding,subtracting,multiplying even and odd numbers.

TEACHING SUGGESTIONS

1. Reinforce meaning of even and odd numbers.

2. Extend to generalization for divisibility by 2.

   Discuss even numbers. Determine rule for divisibility by 2.
   A number is divisible by 2 if and only if the number formed by the digit in the units place is divisible by 2.

3. Addition and Subtraction with Even and Odd Numbers.

   a. Write 2 mathematical sentences to show whether an even number or an odd number results when:

      An even number is added to an even number.

      An even number is subtracted from an even number.

      An odd number is added to an even number.

      An odd number is subtracted fron an even number.

      An even number is subtracted from an odd number.

      An odd number is added to an odd number.

      An odd number is subtracted from an odd number.

b. Fill in the blanks with the word odd and even to make a true statement.

The sum of two even numbers is always an __________ number.

The sum of two odd numbers is always an __________ number.

The sum of an odd and even number is always an _______ number.

c. Fill in these tables e.g. since Odd + Odd = Even we fill in an E in the circled box.

+	O	E
O	Ⓔ	
E		

−	O	E
O		
E		

4. Multiplication with Even and Odd numbers.

Have children draw the following charts.

A

X	1	3	5	7
1				
3				
5				
7				

B

X	2	4	6	8
2				
4				
6				
8				

C

X	2	4	6	8
1				
3				
5				
7				

D

X	1	3	5	7
2				
4				
6				
8				

Children note that:

Chart A has 2 sets of odd numbers.
Chart B has 2 sets of even numbers.
Charts C and D have 1 set of even numbers and 1 set of odd numbers.

They complete each chart by recording all products.

Children complete the following statements:

The product of 2 even numbers is always an _________ number.
The product of 2 odd numbers is always an _________number.
The product of an odd and an even number is always an ___________ number.

Children should generalize by completing and interpreting an array such as the following:

X	odd	even
odd	[odd]	[even]
even	[even]	[even]

X	O	E
O	O	E
E	E	E

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

1. If the replacement set for n is the set of odd numbers, write all the values of n to make the following sentences true:

   $n \times 8 < 50$ $n \times 3 < 30$

   $7 \times n < 49$ $n \times 9 < 81$

2. If the replacement set for n is the set of even numbers, write three values of n to make the following sentences true:

   $n \times 6 < 50$ n has 3 as a factor

   $3 \times n$ is an even number n has 7 as a factor

3. Write three illustraions for each sentence below using numerals. Tell whether the product will be odd or even.

   even times odd odd times odd

   even times even odd times even

4. Write five even multiples of 3. Write four even multiples of 9.
   Write five even multiples of 8. Write four odd multiples of 5.
   Write five even multiples of 7. Write four odd multiples of 7.

5. Can a multiple of 8 ever be an odd number? Discuss.

6. Can a multiple of any even number be an odd number?

7. Can a multiple of any odd number be an even number?

8. Can the sum of 4 odd numbers be an odd number?

*9. Have children consider the set of even numbers { 0,2,4,6, ... }

Which of the following properties are true for this set? Give an example of each.

The Commutative Property for Addition.

The Associative Property for Addition.

Closure for Addition.

The Commutative Property for Multiplication.

The Distributive Property for Multiplication.

Closure with respect to Multiplication.

*10. Have children consider the Set of Odd Numbers and observe that:

Closure for Addition does not appply to this set of numbers.

[3 + 5 = 8]

Multiplication is closed within this set.

OPERATIONS

UNIT 55 - MULTIPLICATION OF WHOLE NUMBERS: EXPLORING PATTERNS

OBJECTIVE: To drill multiplication facts and the extension of facts by the use of patterns.

TEACHING SUGGESTIONS

Patterns For Multiplication Facts

The discovery of patterns by children helps them to see relationships and gives them insight into mathematical structure.

Patterns as Shown in the Multiplication Table

1. Have children use rexographed outlines or graph paper to organize a multiplication "table" as shown below:

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6							27
4	0	4								36
5										45
6										54
7										63
8										72
9										81

Include:
The operational symbol "x"; 0 to 9 in the left hand column;
0 to 9 along the top row.

2. Ask children to complete one row at a time as shown above and to observe patterns that show differences between consecutive numbers.

   What are the differences between two consecutive numbers in each vertical column?

   What is the relationship between the differences and the heading?

3. Ask children to examine the product in any one column to note:

   Commutativity: Show how the product of any two factors is not affected by the order of the factors. [8 x 7 = 7 x 8, etc.]

   Zero in Multiplication: n x 0 = 0
   Note the horizontal and vertical rows of zero.
   Why are there complete lines of zero.

   Property of "1" in Multiplication: n x 1 = n
   Note the numerals in the second rows, and compare these with the heading row. Why are they the same?

   Children should note the numerals in the second column and compare these with the numerals at the left. Why are these the same?

   Pattern Showing Factors:
   Note that 12 appears four times. Why?
   30 appears only twice. Why?

   Find the numerals that appear only once. Why?

Patterns for Multiples of 9

1. Have children name the multiples of 9, beginning with 9. Teacher should record: {9, 18, 27 ...}

   How much is added each time?

   How do the digits in the one's place change? Why?

   How do the digits in the ten's place change? Why?

2. Ask children to note the sum of the digits as they add the ones and tens digit in each multiple of 9. For example:

$9: \; 0 + 9 = 9$
$18: \; 1 + 8 = 9$
$27: \; 2 + 7 = 9$

What is the sum of the digits each time?

3. Write 3 or more equations in the following sequences:

$1 \times 9 = (1 \times 10) - 1 = n$ Why do we subtract 1?
$2 \times 9 = (2 \times 10) - (2 \times 1) = n$ Why do we subtract 2?
$3 \times 9 = (3 \times 10) - 3 = n$ Why do we subtract 3?

What is the pattern in the successive numbers that are subtracted from 10, 20, 30, etc.?

4. Ask children to find the product of 17 x 9; 32 x 9; 163 x 9. Add the ones, tens, hundreds digits of each product.

What is the sum?

What will the sum be if you add the ones, tens, etc. digits of any multiple of 9? [a multiple of 9]

Generalization: Have children tell how they will know if a number is divisible by 9.

5. Have children make the following chart from 1 to 100.

1	2	3	4	5	6	7	8	⑨	10
11	12	13	14	15	16	17	⑱	19	20
21	22	23	24	25	26	㉗	28	29	30
31	32	33	34	35	㊱	37	38	39	40
41	42	43	44	㊺	46	47	48	49	50
51	52	53	(54)	55	56	57	58	59	60
61	62	(63)	64	65	66	67	68	69	70
71	(72)	73	74	75	76	77	78	79	80
(81)	82	83	84	85	86	87	88	89	(90)
91	92	93	94	95	96	97	98	(99)	100

Circle all the multiples of 9. Observe the pattern.

Why does the encircled numeral move 1 column to the left each time?

Why is "90" an exception?

Compare each multiple of 9 with the next whole-decade number:

9 with 10; 18 with 20; 27 with 30; etc.

(1×9 is 1 less than 10; 2×9 is 2 less than 20; etc.)

Patterns for Multiples of 8

1. Have children name the multiples of 8, beginning with 8.
   Teacher should record: {8, 16, 24 ...}

   By how much does the units digit decrease each time? Why?

2. Write 3 or more equations in the following sequences:

   $1 \times 8 = (1 \times 10) - 2 = 10 - 2 = n$
   $2 \times 8 = (2 \times 10) - (2 \times 2) = 20 - 4 = n$
   $3 \times 8 = (3 \times 10) - (3 \times 2) = 30 - 6 = n$

   [Note the use of the Distributive Property]

   What is the pattern in the successive numbers that are subtracted from 10, 20, 30, etc. to arrive at multiples of 8?

3. Have children make a number chart from 1 to 100 as for multiples of 9. Circle the multiples of 8.
   They should observe the pattern and discuss:

   In what columns are there no multiples of 8? Why?

   In each column what is the difference between the successive multiples?

Patterns for Multiples of Other Numbers

Ask children to make other number charts from 1 to 100.
They block out the multiples of 3; of 7; etc.
Children should try to discover patterns and discuss what they find.

Sequence Showing Patterns

Ask children to find the pattern in the sequences below and write the next two numbers:

1, 2, 4, 8, 16, __, __, [Power of 2]

1, 3, 9, 27, __, __, [Power of 3]

1, 2, 6, 24, __, __, [1, 1×2, $1 \times 2 \times 3$, $1 \times 2 \times 3 \times 4$, etc.]

SUGGESTED EXERCISES

1. Write a complicated addition fact that can be solved by multiplication.

2. Make a "table" of Multiplication Facts and answer the questions below.

   Why are all the numerals in the second vertical column the same as the numerals at the left side?

   Why are all the numerals in the second vertical row the same as the numerals across the top of the chart?

   The product of any number and 1 is ——? [Identity Element]

   Most of the numerals in the multiplication chart are repeated at least 2 times. Why? (Commutativity)

   When two whole numbers are multiplied, what kind of number will the product always be? [Whole Number] (Property of Closure)

3. Write three numerical sentences to illustrate the statement:

   If you change the order of the factors you do not change the value of the product. (Commutativity)

4. Answer the following:

   Is every multiple of 6 also a multiple of 3? [Yes] Why?

   Is every multiple of 3 also a multiple of 6? [No] Why?

   One factor of 24 is 8. Write the other factor.

   Factor 24 in different ways.

5. Complete each of the following open sentences to make the statements true.

7 x 4 = 4 x □	8 x 9 = 36 + □
6 x □ = 5 x 6	5 x 7 = (4 x 7) + □
	7 x 9 = 27 + 27 + □
3 x 10 = 6 x □	9 x 6 = (10 x 6) − □
6 x 4 = 12 x □	9 x 9 = 90 − □

6. Examine each of the following equations and answer the questions below:

$$7 \times 9 = (4 \times 9) + (3 \times 9)$$
$$7 \times 9 = (5 \times 9) + (2 \times 9)$$
$$7 \times 9 = (6 \times 9) + (1 \times 9)$$

How did you rename 7×9 in each equation?

What was the product each time?

How did renaming the 7 affect the product?

How can you find a product that you may not know?

What property is being used? [Distributive]

7. Solve the following open sentences in as many ways as you can:

$$\begin{aligned} 7 \times 9 &= (3 + 4) \times 9 = (\square \times 9) + (\triangle \times 9) = n \\ &= (5 + \triangle) \times 9 = (5 \times 9) + (\triangle \times 9) = n \\ &= (6 + \triangle) \times 9 = (6 \times 9) + (\triangle \times 9) = n \end{aligned}$$

$$\begin{aligned} 16 \times 7 &= (\square + \square) \times 7 = (\square \times 7) + (\square \times 7) = n \\ &= (9 + \triangle) \times 7 = (9 \times 7) + (\triangle \times 7) = n \\ &= (7 + \triangle) \times 7 = (7 \times 7) + (\triangle \times 7) = n \end{aligned}$$

$$\begin{aligned} 6 \times 8 &= (n \times 8) + (n \times 8) \\ &= (4 \times 8) + (n \times 8) \\ &= (5 \times 8) + (n \times 8) \end{aligned}$$

$$\begin{aligned} 14 \times 8 &= (n \times 8) + (n \times 8) \\ &= (9 \times 8) + (n \times 8) \\ &= (8 \times 8) + (n \times 8) \end{aligned}$$

8. Find the largest value for n that will make the sentence true, if the replacement set = the set of whole numbers.

$n \times 8 < 17$

$n \times 8 < 34$

$n \times 8 < 45$

$n \times 8 < 85$

$n \times 8 < 25$

$n \times 8 < 50$

$n \times 8 < 38$

$n \times 8 < 69$

$n \times 8 < 30$

$n \times 8 < 60$

$n \times 8 < 23$

$n \times 8 < 55$

* 9. Just For Fun (Puzzles generally intrigue children)

Have children:

Choose any one of the following numbers:

6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

Multiply the number by 2.

Now add 10.

Take one half of the sum.

Subtract from this, the number originally selected.

What is your answer? [5]

Can you explain why the answer will always be 5.

$$\left[\begin{array}{l} \dfrac{2n + 10}{2} = n + 5 \\ \\ (n + 5) - n = 5 \end{array}\right]$$

Make the same puzzle using other numbers and following the same pattern of operations.

OPERATIONS

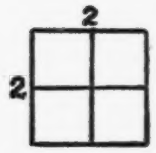
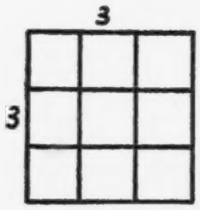
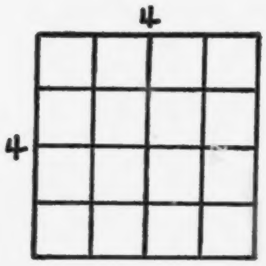
UNIT 56 - MULTIPLICATION OF WHOLE NUMBERS : SQUARES OF NUMBERS

Objective: To develop understanding of squares of whole numbers.

TEACHING SUGGESTIONS

1. Begin by using squares to show patterns.

   Have children use unruled paper or graph paper to mark off and count the number of boxes in a:

2 by 2 square or 2 x 2	3 by 3 square or 3 x 3	4 by 4 square or 4 x 4
		

Tell children that another way of referring to 2 twos, 3 threes, 4 fours is 2 squared or 2 square, 3 squared or 3 square, 4 squared or 4 square.

Why are the products 4, 9, 6 referred to as squares of 2, 3, 4?

Have children observe that 16 is also the product of 8 x 2, but it is only because a number is the product of the same factor used two times that it is considered a perfect square.

Have children illustrate that the representation of 4 x 4 as an array results in a perfect square, while the representation of 8 x 2 as an array does not.

• • • •   • • • • • • • •
• • • •   • • • • • • • •
• • • •
• • • •

2. Children make diagrams to show the squares of other numbers:

The square of 5;   6 squared;   7 x 7;   etc.

Have children observe that 1, 4, 9, 16 etc. are called "squares" because they represent the number of elements in square arrays.

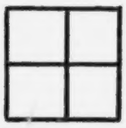
4 may be represented as ⁚ ⁚ ;

9 as • • • ; etc.
• • •
• • •

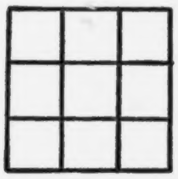
Children also observe that:

A 1 by 1 square contains 1 box

A 2 by 2 square contains 4 boxes



A 3 by 3 square contains 9 boxes



etc.

3. Children should draw squares like the ones below and fill in the multiplication facts.

For a 2 x 2 square

x	1	2
1	1	2
2	2	4

For a 4 x 4 square

x	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

For a 5 x 5 square

x	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

Ask children to count the number of boxes in a 2 x 2 square; a 4 x 4 square, etc.
They observe that a 2 by 2 square contains 2 x 2 = 4 boxes; a 4 by 4 square contains 4 x 4 = 16 boxes; etc.

Ask children to draw diagonals of the squares in each array. They should note that all the squared numbers lie along one diagonal. Why?

SUGGESTED EXERCISES

1. Solve and construct diagrams to complete the solutions.

   The square of 6 = □ x □ = △
   The square of 8 = □ x □ = △
   7 squared = □ x □ = △
   9 squared = □ x □ = △
   1 squared = □ x □ = △

   4 = n squared n = ?
   9 = n squared n = ?
   16 = n squared n = ?
   25 = n squared n = ?
   36 = n squared n = ?

2. Which of the following are squares? Of what numbers?

   7, 9, 54, 64, 19, 49, 80, 81

3. What are the equal factors that result in the following squares?

   25, 36, 64, 4, 9

4. Find the square of each number:

   3, 5, 7, 9, 2, 4, 6, 8, 0, 1

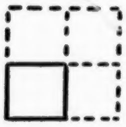
5. 36 is the product of 2 x 18, and of 6 x 6. Which pair of factors can be represented by an array to show that 36 is a square? Why?

*6. Interesting Numbers (Optional)

Start with a 1 unit square. □

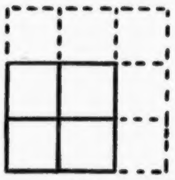
How many unit squares must we add to the 1 unit square to make a 2 by 2 square? [3]

Show this in a diagram.



Show this as a sum. [1 + 3 = 2 squared = 4]

Extend the 2 by 2 square to a 3 by 3 square.



How many units did you add? [5]

Show this as a sum. [1 + 3 + 5 = 3 squared = 9]

Extend the 3 by 3 square to a 4 by 4 square.

What pattern of numbers do you observe?

$$1 + 3 = 2 \text{ squared} = 4$$

$$\underbrace{1 + 3}_{2 \text{ squared}} + 5 = 3 \text{ squared} = 9$$

$$\underbrace{1 + 3 + 5}_{3 \text{ squared}} + 7 = 4 \text{ squared} = 16$$

What do you observe about the number that is added each time?

Can you continue the pattern for a 5 by 5 square? for a 6 by 6 square?

OPERATIONS

UNIT 57 - DIVISION OF WHOLE NUMBERS: PROPERTIES OF "1" AND ZERO

Objectives: To develop an understanding of the property of "1" in division.

To compare the properties of "1" in all operations.

To help children arrive at generalizations for divisor, dividend, quotient relationshops.

To develop an understanding of the role of zero in division.

TEACHING SUGGESTIONS

Property of "1" in Divisor

Quotient "1"

1. In each of the following exercises have children complete the open sentence.

[] x 7 = 7 $\begin{array}{r}\square\\ 7\overline{)7}\end{array}$ [] x 35 = 35 $\begin{array}{r}\square\\ 35\overline{)35}\end{array}$

[] x 8 = 8 $\begin{array}{r}\square\\ 8\overline{)8}\end{array}$ $\square \times n = n$ $\begin{array}{r}\square\\ n\overline{)n}\end{array}$

[] x 9 = 9 $\begin{array}{r}\square\\ 9\overline{)9}\end{array}$

2. Discuss and state the generalization:

Any number, except zero, divided by itself equals 1.

Therefore, $n \div n = 1$ if $n \neq 0$.

Divisor "1"

1. Present open sentences in which "1" is a given factor.

Children solve and discuss each quotient.

$\square \times 1 = 7$ $\quad$ $\begin{array}{r}\square\\ 1\overline{)\,7}\end{array}$ $\quad$ $\square \times 1 = 35$ $\quad$ $\begin{array}{r}\square\\ 1\overline{)\,35}\end{array}$

$\square \times 1 = 8$ $\quad$ $\begin{array}{r}\square\\ 1\overline{)\,8}\end{array}$ $\quad$ $\square \times 1 = n$ $\quad$ $\begin{array}{r}\square\\ 1\overline{)\,n}\end{array}$

2. Develop, then state the generalization.

   Any number, divided by 1 results in the same number.

   Therefore, $\frac{n}{1} = n$

   Ask children: "Do we have to be careful about zero in this case"? [No]

Comparing Properties of "1" in all Operations for the Set of Whole Numbers

1. Have a chart similar to the one below.
   Develop:

Addition	Subtraction	Multiplication	Division
$8 + 1 = 9$	$8 - 1 = 7$	$8 \times 1 = 8$	$8 \div 1 = 8$
$1 + 8 = 9$	$1 - 8$ (Subtraction is not always possible in the set of whole numbers)	$1 \times 8 = 8$	$1 \div 8$ (Division is not always possible in the set of whole numbers)

2. Discuss and compare properties of "1" with respect to the different operations.

   What happens when:

   1 is added to 8? $\quad$ 8 is added to 1?
   1 is multiplied by 8? (8x1) $\quad$ 8 is multiplied by 1? (1x8)
   1 is subtracted from 8? $\quad$ 8 is divided by 1?

3. Have children compare the role of 1 with respect to:

   Addition vs. Subtraction
   Multiplication vs. Division

   Decide when the results are the same as the original number and when they are different.

4. Discuss the role of 1 as an "Identity" element in multiplication.

5. Compare with 0 for addition.

$$\begin{bmatrix} a \times 1 = 1 \times a = a \\ a + 0 = 0 + a = a \end{bmatrix}$$

Divisor - Dividend - Quotient Relationship

Present sets of equations. Have children compare dividends, divisors, and quotients and state the relationships for each set, as follows:

1. Doubling the Dividend: Keeping the Divisor Constant

$\begin{array}{r} 1 \\ 6\overline{)6} \end{array}$	$6 \div 6 = 1$	$\begin{array}{r} 3 \\ 8\overline{)24} \end{array}$	$24 \div 8 = 3$
$\begin{array}{r} \square \\ 6\overline{)12} \end{array}$	$12 \div 6 = \square$	$\begin{array}{r} \square \\ 8\overline{)48} \end{array}$	$48 \div 8 = \square$
$\begin{array}{r} \square \\ 6\overline{)24} \end{array}$	$24 \div 6 = \square$	$\begin{array}{r} \square \\ 8\overline{)96} \end{array}$	$96 \div 8 = \square$
$\begin{array}{r} \square \\ 6\overline{)48} \end{array}$	$48 \div 6 = \square$	$\begin{array}{r} \square \\ 8\overline{)192} \end{array}$	$192 \div 8 = \square$

Develop the generalization:

If the dividend is doubled and the divisor remains the same, the quotient is doubled.

2. Dividing the Dividend by 2: Keeping the Divisor Constant

$\begin{array}{r} \square \\ 6\overline{)24} \end{array}$	$24 \div 6 = \square$	$\begin{array}{r} \square \\ 8\overline{)48} \end{array}$	$48 \div 8 = \square$
$\begin{array}{r} \square \\ 6\overline{)12} \end{array}$ Why?	$12 \div 6 = \square$	$\begin{array}{r} \square \\ 8\overline{)24} \end{array}$ Why?	$24 \div 8 = \square$
$\begin{array}{r} \square \\ 6\overline{)6} \end{array}$	$6 \div 6 = \square$	$\begin{array}{r} \square \\ 8\overline{)12} \end{array}$	$12 \div 8 = \square$
$\begin{array}{r} \square \\ 6\overline{)3} \end{array}$	$3 \div 6 = \square$	$\begin{array}{r} \square \\ 8\overline{)6} \end{array}$	$6 \div 8 = \square$

Ask this question:

> If a dividend is divided by 2 and the divisor remains the same, how has the quotient changed?

Have the children state the result as a generalization.

3. Doubling the divisor: Keeping the Dividend Constant

$24 \div 3 = 8$
$24 \div 6 = \square$ Why?
$24 \div 12 = \square$
$24 \div 24 = \square$

$68 \div 4 = 17$
$68 \div 8 = \square$ Why?
$68 \div 16 = \square$
$68 \div 32 = \square$

Ask this question:

> If a divisor is doubled and the dividend remains the same, how has the quotient changed?

4. Dividing the Divisor by 2: Keeping the Dividend Constant

$24 \div 24 = 1$
$24 \div 12 = \square$ Why?
$24 \div 6 = \square$
$24 \div 3 = \square$

$120 \div 120 = 1$
$120 \div 60 = \square$ Why?
$120 \div 30 = \square$
$120 \div 15 = \square$

If a divisor is divided by 2 and the dividend remains the same, how has the quotient changed?

5. Doubling Both Dividend and Divisor

$6 \div 3 = 2$
$12 \div 6 = \square$ Why?
$24 \div 12 = \square$
$48 \div 24 = \square$

$18 \div 6 = 3$
$36 \div 12 = \square$
$72 \div 24 = \square$
$144 \div 48 = \square$

If a divisor is doubled and the quotient remains the same, how has the dividend changed?

If a dividend is doubled and the quotient remains the same, how has the divisor changed?

If both dividend and divisor are doubled, how has the quotient changed?

6. **Dividing Both Dividend and Divisor by 2**

64 ÷ 16 = 4	240 ÷ 80 = 3	480 ÷ 120 = 4
32 ÷ 8 = □ Why?	120 ÷ 40 = □ Why?	240 ÷ 60 = □ Why?
16 ÷ 4 = □	60 ÷ 20 = □	120 ÷ 30 = □
8 ÷ 2 = □	30 ÷ 10 = □	60 ÷ 15 = □

If a divisor is divided by 2, and the quotient remains the same, how has the dividend changed?

If a dividend is divided by 2, and the quotient remains the same, how has the divisor changed?

If both the dividend and divisor are divided by 2, how has the quotient changed?

Zero in Division

Zero Divided by a number (Zero as the dividend)

1. Present a series of divisions and their related multiplications.

3 ÷ 1 = n	n x 1 = 3
2 ÷ 1 = n	n x 1 = 2
1 ÷ 1 = n	n x 1 = 1
0 ÷ 1 = n	n x 1 = 0

Ask children:

How many ones are there in three? in two? in one?
How many ones are there in zero?

2. Continue with:

0 ÷ 2 = n	n x 2 = 0
0 ÷ 3 = n	n x 3 = 0
0 ÷ 4 = n	n x 4 = 0

How many twos are there in zero?
What was the value of n that made n x 1 = 0 true? n x 2 = 0? n x 3 = 0?

3. Write the following open sentences involving division, as equivalent open sentences involving multiplication and solve for n.

0 ÷ 4 = n (n x 4 = 0; n = 0)	0 ÷ 5 = n
0 ÷ 9 = n	0 ÷ 8 = n
0 ÷ 17 = n	0 ÷ 19 = n
0 ÷ 35 = n	0 ÷ 63 = n

> If the dividend is zero and the divisor is any non-zero number, what will the quotient be?

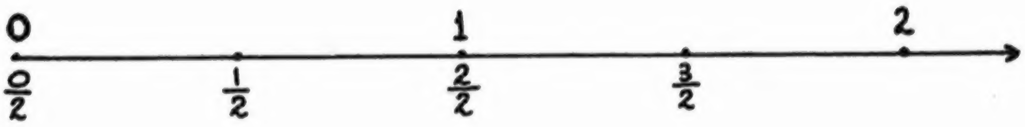
Have children observe that we are considering non-zero divisors. Zero as a divisor is a special case which will be considered later.

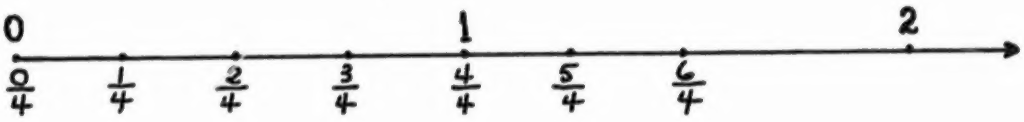
Have children state the generalization:

> If zero is divided by any non-zero number, the quotient is zero.

4. Relate to fraction form. Use number rays.

a.





b. Show on a number ray 3 other ways of representing zero as a fraction.

$$\left[\frac{0}{3}, \ \frac{0}{5}, \ \frac{0}{8} - - - \right]$$

SETS; NUMBER; NUMERATION

UNIT 58 - SET OF FRACTIONAL NUMBERS: CONCEPTS OF FIFTHS AND TENTHS; COMMON FRACTIONAL FORM

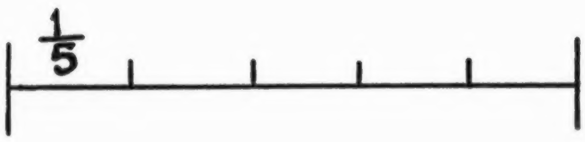
Objectives: To strengthen the concept of fifths and tenths.

To help children understand the relationship of tenths to one; tenths to tenths; tenths to fifths.

TEACHING SUGGESTIONS

Meaning of Fifths

1. Have children draw a line segment indicating one unit. The whole is then divided into fifths (5 equal parts).



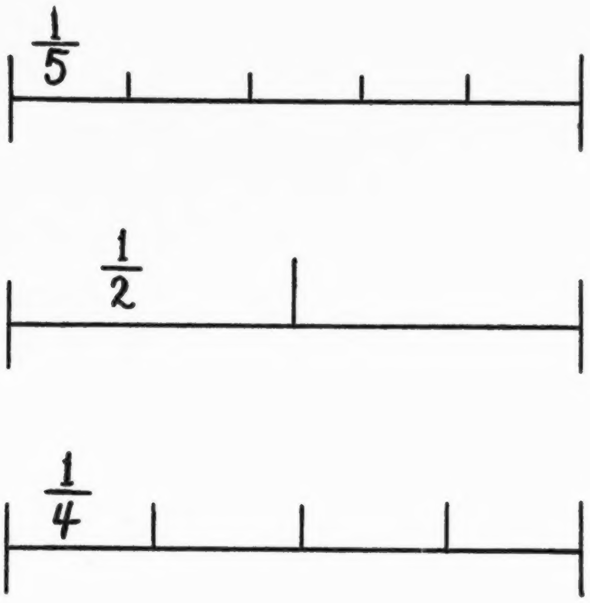
Have children discover that:

Fifths cannot be derived from halves, fourths, eighths, thirds, sixths.

Fifths are so named when a unit has been divided into 5 equal parts.

2. Comparing fifths with: halves, fourths, eighths, thirds, sixths.

   a. Children should draw several lines of equal length.



Which fractional parts are larger than $\frac{1}{5}$? Smaller?

b. Draw 3 lines of equal length.

On segment A locate $\frac{1}{2}$; $\frac{1}{5}$

On segment B locate $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$

On segment C locate $\frac{1}{3}$; $\frac{1}{5}$

Similarly compare: fifths and sixths; fifths and eighths.

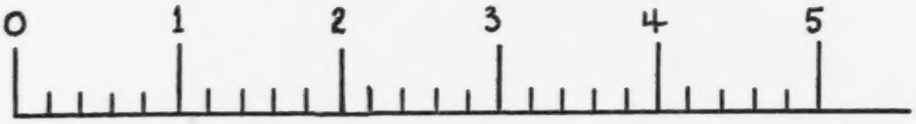
c. Ask children to insert the correct symbol (< , >) to compare each pair of the following fractional parts.

$\frac{1}{5}$ — $\frac{1}{4}$ $\frac{1}{2}$ — $\frac{1}{5}$

$\frac{2}{5}$ — $\frac{1}{4}$ $\frac{3}{5}$ — $\frac{3}{4}$ etc.

3. Equivalents

Draw a number line divided into fifths that includes more than 1 unit, thus:



Have children locate the following: $1\frac{1}{5}$, $1\frac{3}{5}$, $4\frac{4}{5}$, etc.

Reinforce concept that these points also represent distances from the zero point.

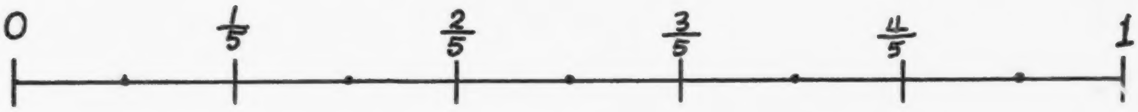
Have children complete the following:

$\frac{5}{5} = \square$ $\frac{6}{5} = \square$ $\frac{7}{5} = \square$ $\frac{12}{5} = \square$ etc.

Concept of Tenths

1. Tenths derived from halves of fifths

Direct children to draw a line segment divided into five equal parts. They then label each fifth, and divide each fifth into 2 equal parts.



What is each part called now?
Label each part.
How much is $\frac{1}{2}$ of $\frac{1}{5}$?
How much is $\frac{1}{2}$ of $\frac{2}{5}$? of $\frac{3}{5}$? of $\frac{4}{5}$? of $\frac{5}{5}$?
Since $\frac{1}{5} = \frac{2}{10}$, $\frac{1}{2}$ of $\frac{1}{5} = \frac{\square}{10}$

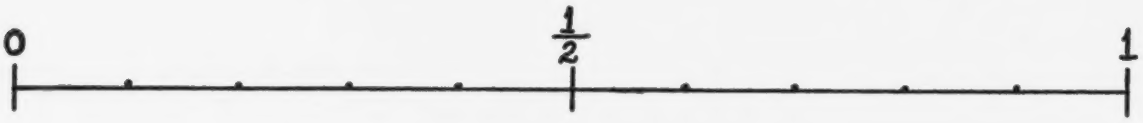
Since $\frac{2}{5} = \frac{4}{10}$, $\frac{1}{2}$ of $\frac{2}{5} = \frac{\square}{10}$

Since $\frac{1}{5} = \frac{2}{10}$, $\frac{1}{10}$ is what part of $\frac{1}{5}$?

$\frac{2}{10}$ is what part of $\frac{2}{5}$?

2. **Tenths derived from fifths of halves.**

Draw a line segment and divide it into 2 equal parts.
Then divide each half into 5 equal parts.



What is each part called now? $\left[\frac{1}{10}\right]$

How much is $\frac{1}{5}$ of $\frac{1}{2}$? Find this length from zero on the line segment.

How much is $\frac{2}{5}$ of $\frac{1}{2}$? Draw an arrow on the line segment to show this.

Have children continue to find fifths of one half in the same way.

Since $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$, $\frac{2}{5}$ of $\frac{1}{2} = \frac{\square}{10}$

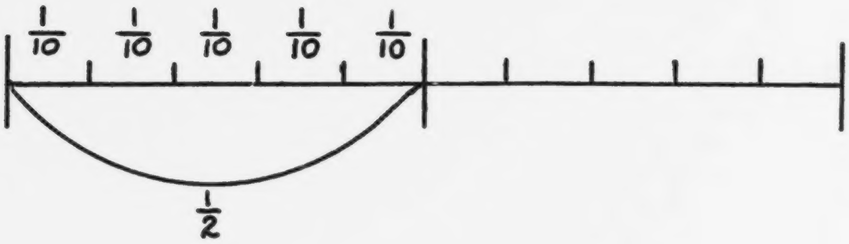
Since $\frac{2}{5}$ of $\frac{1}{2} = \frac{2}{10}$, $\frac{4}{5}$ of $\frac{1}{2} = \frac{\square}{10}$

3. Equivalent Fractions

Use number lines.

Have children complete the following:

Tenths and Halves


$\frac{1}{10}$
$\frac{1}{10}$
$\frac{1}{10}$
$\frac{1}{10}$
$\frac{1}{10}$
$\frac{1}{2}$

$\frac{5}{10} = \frac{\square}{2}$

$\frac{6}{10} = \frac{1}{2} + \frac{\square}{10}$

$\frac{7}{10} = \frac{1}{2} + \frac{\square}{10}$

$\frac{8}{10} = \frac{1}{2} + \frac{\square}{10}$

$\frac{9}{10} = \frac{1}{2} + \frac{\square}{10}$

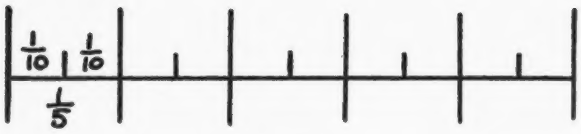
$\frac{10}{10} = \frac{1}{2} + \frac{\square}{10}$

$\frac{10}{10} = \square$

$\frac{10}{10} = \frac{\square}{2}$

Since $\frac{5}{10} = \frac{1}{2}$, $\frac{6}{10} = \frac{1}{2} + \frac{\square}{10}$ and $\frac{8}{10} = \frac{1}{2} + \frac{\square}{10}$

Tenths and Fifths


$\frac{1}{10}$
$\frac{1}{10}$
$\frac{1}{5}$

$\frac{2}{10} = \frac{\square}{5}$

$\frac{4}{10} = \frac{\square}{5}$

$\frac{6}{10} = \frac{\square}{5}$

$\frac{8}{10} = \frac{\square}{5}$

$\frac{10}{10} = \frac{\square}{5}$

$\frac{3}{10} = \frac{\square}{5} + \frac{1}{10}$

$\frac{5}{10} = \frac{\square}{5} + \frac{1}{10}$

$\frac{7}{10} = \frac{\square}{5} + \frac{1}{10}$

$\frac{9}{10} = \frac{\square}{5} + \frac{1}{10}$

Compare tenths with: halves, fourths, eighths, thirds, sixths, fifths using line diagrams.

4. Counting

Use line diagrams. Have children record fractions on the number line as they count.

a. Counting forward - groups of $\frac{2}{5}$

$\frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \ldots$ then $\frac{2}{5}, \frac{4}{5}, 1\frac{1}{5}, \ldots \left[1\frac{3}{5}, 2, 2\frac{2}{5}\right]$

$\frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \ldots$ then $\frac{3}{5}, 1, 1\frac{2}{5}, \ldots \left[1\frac{4}{5}, 2\frac{1}{5}, 2\frac{3}{5}\right]$

b. Counting backward - groups of $\frac{3}{5}$

$\frac{13}{5}, \frac{10}{5}, \frac{7}{5}, \ldots$ then $\frac{13}{5}, 2, \frac{7}{5}, \ldots \left[\frac{4}{5}, \frac{1}{5}\right]$

then $2\frac{3}{5}, 2, \ldots \left[1\frac{2}{5}, \frac{4}{5}, \frac{1}{5}\right]$

c. Counting forward

$\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \ldots$ then $\frac{1}{10}, \frac{3}{10}, \frac{1}{2}, \frac{7}{10}, \frac{9}{10}, \ldots$

$\frac{1}{10}, \frac{3}{10}, \frac{5}{10}, \frac{7}{10}, \frac{9}{10}, \ldots$ then $\frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \ldots$

d. Counting backward

$\frac{11}{10}, \frac{10}{10}, \frac{9}{10}, \frac{8}{10}, \frac{7}{10}, \ldots$ then $1\frac{1}{10}, 1, \frac{9}{10}, \frac{4}{5}, \frac{7}{10}, \ldots$

$\frac{9}{10}, \frac{7}{10}, \frac{5}{10}, \frac{3}{10}, \frac{1}{10}, \ldots$ then $\frac{9}{10}, \frac{7}{10}, \frac{1}{2}, \frac{3}{10}, \frac{1}{10}, \ldots$

e. Additional Suggestions.

Have children change to simplest form as they count.

Count by $\frac{3}{5}$ starting with $1\frac{3}{5}$

Count by $\frac{3}{10}$ starting with $1\frac{1}{2}$

Count backward by $\frac{2}{5}$ starting with 2

Count backward by $\frac{3}{10}$ starting with 2

Find the distance between the points.

$\frac{4}{5}$ and $1\frac{1}{5}$; $1\frac{1}{5}$ and $2\frac{1}{5}$; $3\frac{2}{5}$ and $4\frac{3}{5}$, etc.

5. Relationship of tenths to one; Tenths to tenths

a. Have children insert the correct numeral to complete the equation below. Refer to a line diagram when necessary.

$\frac{2}{10} = \square$ times $\frac{1}{10}$ $\frac{3}{10} = \square$ times $\frac{1}{10}$ $\frac{6}{10} = \square$ times $\frac{2}{10}$ etc.
$\frac{10}{10} = \square$ times $\frac{1}{10}$ $\frac{10}{10} = \square$ times $\frac{2}{10}$ $\frac{10}{10} = \square$ times $\frac{5}{10}$
$1 = \square$ times $\frac{1}{10}$ $1 = \square$ times $\frac{2}{10}$ $1 = \square$ times $\frac{5}{10}$

b. Solve the following problems:

$\frac{1}{10}$ is what part of: $\frac{2}{10}$, $\frac{3}{10}$, — — — $\frac{10}{10}$, 1 ?
$\frac{2}{10}$ is what part of: $\frac{4}{10}$, $\frac{6}{10}$, $\frac{8}{10}$, $\frac{10}{10}$, 1 ?

c. What is the relationship between:

$\frac{1}{10}$ and $\frac{10}{10}$; $\frac{10}{10}$ and $\frac{1}{10}$; $\frac{1}{10}$ and 1; 1 and $\frac{1}{10}$

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Use the symbol > to show which fraction of each pair is greater.

   $\frac{2}{3}$ or $\frac{4}{5}$ $\qquad$ $\frac{5}{10}$ or $\frac{3}{5}$ $\qquad$ $\frac{10}{10}$ or 2

2. Which is smaller? Use the symbol < to show this.

   $\frac{3}{5}$ or $\frac{3}{8}$ $\qquad$ $\frac{2}{5}$ or $\frac{2}{10}$ $\qquad$ $\frac{5}{6}$ or $\frac{5}{10}$

3. Which is greater? Explain.

   $\frac{3}{4}$ or $\frac{3}{5}$ $\qquad$ $\frac{5}{6}$ or $\frac{5}{8}$

4. Count forward. Fill in the missing fractions.

   $\frac{1}{10}, \frac{1}{5}, \_\_, \frac{2}{5}, \_\_, \frac{3}{5}, \_\_$

5. Draw a number line and show that:

   $\frac{3}{5} < \frac{7}{10}$; $\qquad$ $\frac{9}{10} < \frac{3}{2}$; $\qquad$ $\frac{8}{10} = \frac{4}{5}$

6. Name the numbers from the smallest to the greatest.

   $\frac{3}{7}, \frac{3}{10}, \frac{3}{5}, \frac{3}{2}$

7. Complete the following:

   $1\frac{5}{10} = 1\frac{\square}{10}$; $\qquad$ $\frac{17}{10} = 1\frac{\square}{10}$; $\qquad$ $\frac{30}{10} = \square$; $\qquad$ $\frac{19}{10} = 1\frac{9}{\square}$

SETS; NUMBER; NUMERATION

UNIT 59 - SET OF FRACTIONAL NUMBERS: TENTHS; DECIMAL FORM

NOTE TO TEACHER

A decimal is a numeral. It is another form for recording a number written as a fraction whose denominator is 10 or a power of 10. Both $\frac{1}{10}$ and .1 are numerals that name the same idea - the fractional number "one-tenth."

Children should understand the relationship of:

Tenths to halves
Tenths to one
Tenths to fifths
One to tenths

Objectives: To extend understanding of multiplicative relationships of place value.

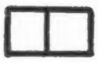
To develop Place Value system; Tenths, decimal form.

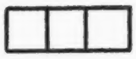
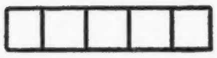
To relate common and decimal forms for tenths.

TEACHING SUGGESTIONS

Multiplicative Relationships: Tenths to 1; One to 10; Ten to 100

1. Discuss experience situations involving tenths: odometer, pedometer.

2. Use squared material to compare a strip of 2 square units  ;

3 square-units  5 square-units 

with 1 square-unit and the reverse.

How many times larger than 1 square-unit is 2 square-units?
3 square-units? etc.

What part of 2 square-units is 1 square-unit? etc.

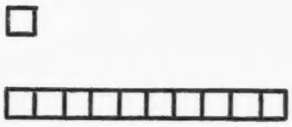
3. Compare a ten-square strip with a one unit-square; a one unit-square with a ten-square strip.

How many times as great as one unit is a ten-square strip?

[10 units is 10 times 1 unit.]

What part of a ten-square strip is one unit?

[1 is $\frac{1}{10}$ of 10]



4. Compare a unit square, a ten square strip and a one hundred square.

Discuss:

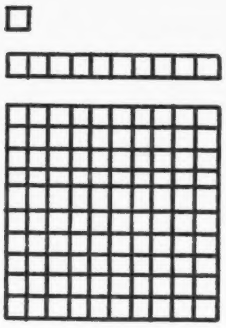
10 to 1 relationship.

10 is how many times 1? [10]
100 is how many times 10? [10]

1 to 10 relationship.

1 is what part of 10? $\left[\frac{1}{10}\right]$

10 is what part of 100? $\left[\frac{1}{10}\right]$



5. Compare with the values of pennies, dimes, and a one-dollar bill.

6. Have children consider 2 unit-squares.

Cut one of the units into 10 approximately equal parts. (▯)

What fractional number does each one of these equal parts represent?

$$\left[\frac{1}{10}\right]$$

Have children compare the uncut unit-square with one of the tenths; with 10 of the tenths.

State the relationships:

10 tenths is equal to 1 unit.

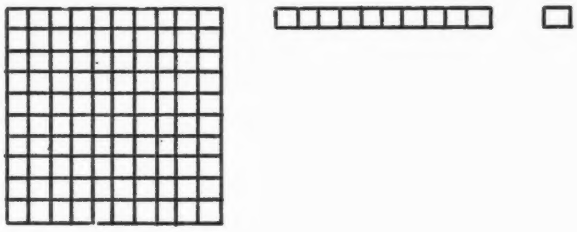
1 unit is 10 times as great as 1 tenth.

1 tenth is $\frac{1}{10}$ of 1 unit.

Numeration System Extended To Tentns: Place Value

1. Place a one-hundred square, a ten-square-strip, and a unit square on the display board. Put a Place-Value Chart directly below the squared material.

Record the numerals for the numbers represented by the material on the Place-Value Chart.



H	T	U
1	1	1

Have available one of the tenths that has been previously cut. (▯)

Ask children:

Where would you place this tenth on the display board?

[right of units place]

Why? Each place has a value one tenth of the place immediately to its left.

Where would you record the numeral for one tenth on the chart?

H	T	U	?
1	1	1	1

What would you call that place? [tenths] Why?

2. Have children label the column "Tenths".

Compare the value of 1 in units place with the value of 1 in tenths place.

H	T	U	Tenths
1	1	1	1

Note:

1 in units place is 10 times the value of 1 in tenths place.

1 in tenths place is $\frac{1}{10}$ the value of 1 in the units place.

3. Have children discuss, then record 2 tenths, 3 tenths, ... 9 tenths.

Where would you record 10 tenths? Why?

Continue to develop understanding of more than 1 unit and tenths.

H	T	U	Tenths
		3	4
		9	1
	2	6	2
1	4	8	3

4. Placing The Decimal Point

Draw a place value chart as shown.

	Units	
2	3	7

Have children label the place to the left of units place. [tens]

Label the place to the right of units place. [tenths]

T	U	Tenths
2	3	7

Have children read the numeral. [23 and 7 tenths]

Erase the labels and lines indicating columns.

> Does the numeral, 237 still indicate 23 and 7 tenths? Explain.
> How can 23 and 7 tenths be shown without labeling each place?

Ask children how tenths are shown on various instruments; odometer, pedometer.

Tell children that another way of recording $23 \frac{7}{10}$ or 23 and 7 tenths is 23.7, which can be read, "twenty-three and seven-tenths".

Discuss the decimal point as a way of identifying units place.

Have children record other numerals. 64.2 29.3 7.8

> What kind of number is represented to the left of the decimal point? [Whole number]
>
> What kind of number is represented to the right of the decimal point? [Fractional number with a value of less than 1]
>
> What do you think 0.1 means? 0.5 means? 0.6 means?
>
> What does the zero to the left of the decimal point indicate?
>
> How does the recording of the decimal fraction differ from recording of the common fraction? [The denominator of the decimal fraction is not written]

5. Suggested Exercises

a. Mark the following statements true or false.

$\frac{3}{10}$ is in decimal form because the denominator is 10.

27.8 is the decimal form of $27\frac{8}{10}$.

.5 has a value of more than 1.

.5 has a value less than 0.

b. Record each of the following as fractions in decimal form.

Three tenths
Five and one tenth
Eleven tenths
Ten tenths

Ninety-eight and six tenths
Three hundred seventy and five tenths
One hundred twenty-three and no tenths

c. Relate the reading and writing of decimal fractions to life situations, such as:
Mileage on R.R. time tables
Temperature and precipitation readings
Records of track meets

Relating Common Form and Decimal Form

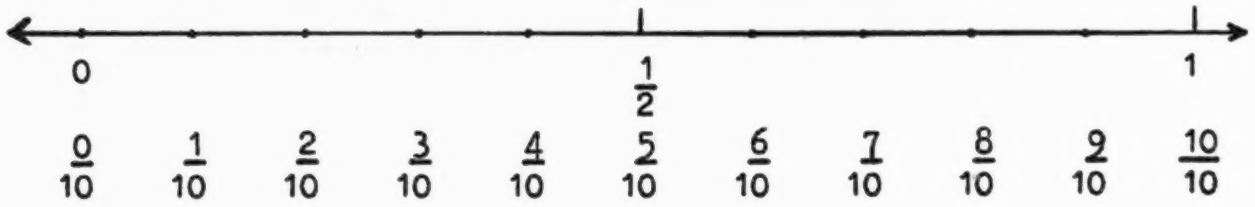
1. Using the Number Line: Halves, Fifths, Tenths

Have children:

Divide one unit of a number line into 2 equal parts.

Label 0, $\frac{1}{2}$, 1 on the number line.

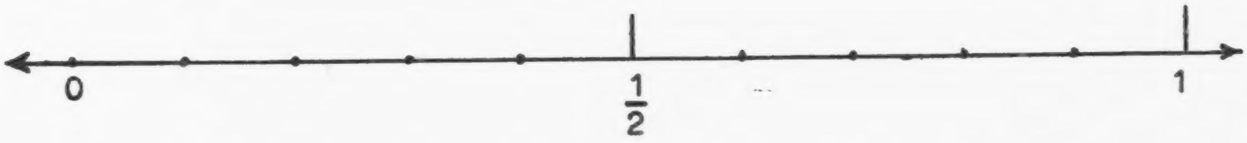
Divide each half into 5 equal parts and label tenths in fractional form.



2. In what other way can we record tenths?

   Have children record decimal form under the fractional form.

   Discuss the equivalent recordings.

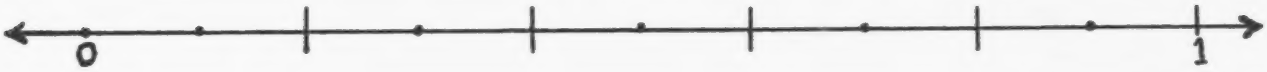


$\frac{0}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$
.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0

. . . .

3. Follow the same procedure dividing the number lines into fifths, then each fifth into 2 equal parts.

   Label each part in both forms.



$\frac{0}{5}$		$\frac{1}{5}$		$\frac{2}{5}$		$\frac{3}{5}$		$\frac{4}{5}$		$\frac{5}{5}$
$\frac{0}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$
.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0

Have children discover:

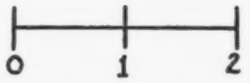
The denominator 10 is not written but is implied by the decimal point when the decimal form is recorded.

Both names $\frac{3}{10}$ and .3 are read in the same way: 3 tenths.

4. Discuss the significance of various numerals on the number line, e.g.

   The numerals $\frac{1}{2}$, $\frac{5}{10}$, .5 all indicate the same distance from zero and name the same number.

5. Extend the lines to represent 2 units of length.



Have children indicate a variety of names for the tenth following 1.0

They may suggest: $\frac{11}{10}$, $1 + \frac{1}{10}$, $1\frac{1}{10}$, $1 + .1$, 1.1, etc.

Continue to name and discuss the meaning of each of the tenths between 1.0 and 2.0.

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Name each of the following in several ways.
   Refer to the number line when necessary.

$\frac{4}{5}$, $\frac{3}{2}$, $\frac{5}{5}$, $\frac{5}{2}$

one half, three fifths, nine fifths, two

$\frac{14}{10}$, $\frac{20}{10}$, $\frac{10}{10}$, $\frac{15}{10}$

2. Insert the correct symbols >, =, <, between each pair of numerals to make a true statement.

$.7$ □ 1 1.2 □ 1 $\frac{10}{10}$ □ 1

1.0 □ 1 $\frac{9}{10}$ □ 1 $\frac{20}{20}$ □ 1

3. Complete the following open sentences.

$.5$ in. $= \frac{1}{\square}$ in. $\frac{23}{\square}$ in. $= 2.3$ in.

2.5 in. = $2\frac{1}{\square}$ in. $\frac{3}{\square}$ in. = .6 in.

$\frac{15}{\square}$ in. = 1.5 in. $1\frac{2}{\square}$ in. = 1.4 in.

4. Write true or false next to each of the following. Explain.

   .8 in. is closer to 1.0 than to .5 in.
   1.4 in. is nearer in value to 1.0 than to 2.0 in.
   2.5 in. is halfway between 2.0 and 3.0 in.

5. Find the number of tenths in 1.0 ft.; in 4.0 ft.; in 10.0 ft.

6. Represent the decimal form of the following sums:

   a. $5 + \frac{3}{10}$

   b. $(4 \times 10) + (7 \times 1) + (5 \times \frac{1}{10})$

   c. $0 + \frac{7}{10}$

7. Write each of the following as the sum of two numbers:

   6.4   14.3   0.1

8. A foot rule calibrated in tenths may be used to:

   Count by tenths.
   Emphasize distances from zero. The numeral 1.5 marks the distance 1.5 units from zero.

Counting

1. Reinforce counting forward. Use the number line when helpful.

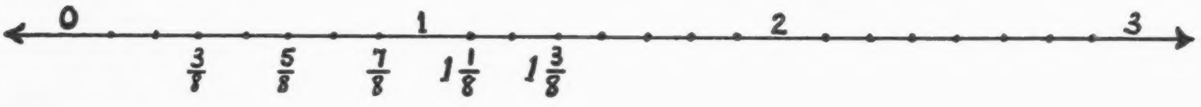
   a. Have children count by steps of 1 half, 1 third, 1 fourth, 1 fifth, etc. beginning with zero. Later they can count using larger intervals. Then count again changing to simpler form.

      For example: $0, \frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}, \frac{10}{5}, \frac{12}{5}, \ldots\ldots$

$$0, \frac{2}{5}, \frac{4}{5}, 1\frac{1}{5}, 1\frac{3}{5}, 2, 2\frac{2}{5}, \ldots\ldots$$

b. Have children count beginning at any point on the number line, counting by jumps of fractions of any size. For example:

Begin at $\frac{3}{8}$, count by jumps of fourths:



c. Discuss sets of fractions in sequence.

When counting by steps of one-tenth: $1\frac{3}{10}$ follows what number? Is followed by what number? $1\frac{2}{5}$ follows what number? etc.

What number is two greater than $3\frac{9}{10}$?

How many tenths must be added to $2\frac{1}{5}$ to reach the next whole number?

d. Have children complete the following sequence:

Count by steps of $\frac{2}{3}$: $3\frac{1}{3}$, 4, $4\frac{2}{3}$, ——, ——, ——, $7\frac{1}{3}$

Count by jumps of $\frac{3}{10}$: $1\frac{1}{5}$, $1\frac{1}{2}$, $1\frac{4}{5}$, ——, ——, ——, 3

Count by steps of .2: 1.3, 1.5, ——, 1.9, ——, 2.3, ——

Count by steps of .3: 7.1, 7.4, 7.7, ——, ——, ——, ——

2. Reinforce counting backward. Use number line when necessary.

a. Children use a number line and record as they count.

They begin with 1 and count backward by intervals of 1 half, 1 third, 1 fourth, 1 fifth, etc. Later using larger intervals.

They count again changing to simpler form.

First: $2,\ 1\frac{7}{9},\ 1\frac{5}{9},\ 1\frac{3}{9},\ 1\frac{1}{9},\ \frac{8}{9},\ \frac{6}{9},\ \frac{4}{9},\ \frac{2}{9},\ 0$

then: $2,\ 1\frac{7}{9},\ 1\frac{5}{9},\ 1\frac{1}{3},\ 1\frac{1}{9},\ \frac{8}{9},\ \frac{2}{3},\ \frac{4}{9},\ \frac{2}{9},\ 0$

b. Children count backward, beginning with any number on the line.

$3\frac{1}{8},\ 2\frac{5}{8},\ 2\frac{1}{8},\ 1\frac{5}{8},\ 1\frac{1}{8},\ \frac{5}{8},\ \frac{1}{8}$ (subtracting $\frac{1}{2}$)

2.1, 1.9, 1.6, 1.3, 1, .7, .4, .1 (subtracting .3)

c. Discuss fractions in sequence:

When counting backward by one-tenth: 1.3 comes before what number?

What number is two less than $3\frac{11}{20}$? than 5.1?

How many twelfths must be subtracted from $2\frac{1}{3}$ to reach the nearest whole number?

OPERATIONS

Unit 60 - SET OF FRACTIONAL NUMBERS: ADDING AND SUBTRACTING TENTHS; DECIMAL FORM; HORIZONTAL AND VERTICAL FORMAT; WITHOUT EXCHANGE

NOTE TO TEACHER

Properties of Operations that apply to fractional numbers in fractional form apply to fractional numbers in decimal form. For addition, the Associative and Commutative properties hold and zero is the Identity Element.

Objective: To add and subtract tenths: (decimal form)by:

Relating common fractional and decimal forms.
Applying The Associative Property of Addition.
Developing vertical algorithms, no exchange.

TEACHING SUGGESTIONS

Suggested experience situations to use in this unit are:

Distances walked.
Distances traveled by bus.
Using a cyclometer, etc.

Horizontal Format

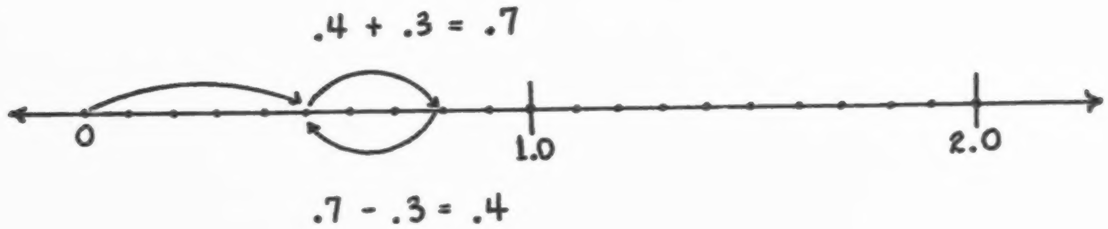
Sums and Minuends Less Than 1

Use number lines to show relationship between adding and subtracting tenths.

1. Begin with addition:

Adding .3 $.4 + .3 = .7$

Subtracting .3 $.7 - .3 = .4$



To add .4 + .3, children find .4 on the number line, then move 3 of the one-tenth spaces to the right.

To subtract .3 from .7, children locate .7 on the number line, then move 3 of the one-tenth spaces to the left.

2. Begin with subtraction:

Subtracting .3 $.7 - .3 = .4$

Adding .3 $.4 + .3 = .7$

3. Have children replace the frame to make true statements.

$.4 + .3 = \frac{4}{10} + \frac{\triangle}{10} = n$ $.7 - .3 = \frac{7}{10} - \frac{\triangle}{10} = n$

$.2 + .5 = \frac{2}{10} + \frac{5}{\triangle} = n$ $.7 - .5 = \frac{7}{10} - \frac{5}{\triangle} = n$

$\frac{7}{10} + \frac{1}{\triangle} = .7 + .1 = n$ $.8 - \triangle = \frac{8}{10} - \frac{1}{10} = n$

$\triangle + .6 = \frac{3}{10} + \frac{6}{10} = n$ $\triangle - .3 = \frac{9}{10} - \frac{3}{10} = n$

Sums and Minuends Greater Than 1

1. Reinforce renaming fractions in decimal form as whole number and/or a whole number plus a fraction.

$.9 + .1 = \square$ $\quad$ $1.0 - .1 = \square$

$.8 + .2 = \square$ $\quad$ $1.0 - .2 = \square$, etc.

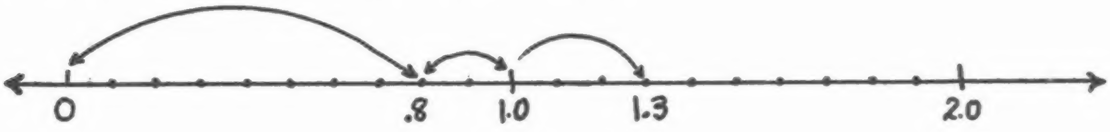
$.1 + .9 = \square$ $\quad$ $1.0 - .9 = \square$

$.2 + .8 = \square$ $\quad$ $1.0 - .8 = \square$, etc.

2. When children find the sum of $.8 + .5$, they may think of $.5$ as $(.2 + .3)$

$.8 + .5 = .8 + (.2 + .3) = (.8 + .2) + .3 = 1 + .3$ or 1.3

(Application of Associative Property)



$.8 + .5 = 1.3$

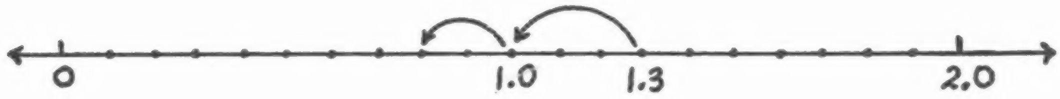
Children may record the sum in several ways.

13 tenths or 1 and 3 tenths or $1.0 + .3$ or 1.3 or $1\frac{3}{10}$

3. $1.3 - .5 = n$

When children find the remainder for $1.3 - .5$, they should think of $.5$ as $(.3 + .2)$.

$(1.3 - .3) - .2 = 1 - .2 = .8$



$1.3 - .5 = .8$

Have children record the remainder in several ways. 8 tenths, $\frac{8}{10}$, .8, etc.

4. Provide practice exercises.

Complete the following

A. $\frac{3}{10} + \frac{7}{10} = \triangle$ $.3 + .7 = \square$

$\frac{5}{10} + \frac{\triangle}{10} = 1$ $.5 + \triangle = 1$

B. $\frac{3}{10} + \frac{8}{10} = \frac{3}{10} + \frac{\triangle}{10} + \frac{1}{10} = \square$ $.3 + .8 = .3 + \quad + .1 = \square$

$\frac{5}{10} + \frac{6}{10} = \frac{5}{10} + \frac{5}{10} + \frac{\triangle}{10} = \square$ $.5 + .6 = .5 + .5 + \quad = \square$

Tell what property helps to solve the problems in B.

Complete the following:

$\frac{10}{10} - \frac{1}{10} = \triangle$ $1.0 - .1 = \triangle$

$\frac{10}{10} - \frac{\triangle}{10} = \frac{6}{10}$ $1.0 - \triangle = .6$

$1\frac{2}{10} - \frac{4}{10} = 1\frac{2}{10} - \frac{2}{10} - \triangle = \square$ $1.2 - .4 = 1.2 - .2 - \triangle = \square$

$1\frac{6}{10} - \frac{8}{10} = 1\frac{6}{10} - \triangle - \frac{2}{10} = \square$ $1.6 - .8 = 1.6 - \triangle - .2 = \square$

Complete the following and then explain the property involved.

$.7 + .6 = \triangle$ and $\triangle - .6 = .7$
$.6 + .7 = \triangle$ and $\triangle - .7 = .6$

Since $.6 + .9 = 1.5$, then $1.5 - .9 = \square$
Since $.9 + .6 = 1.5$, then $1.5 - .6 = \square$

Vertical Format

1. Addition: No Exchange

Suggested Problem: During April, the average rainfall was 5.3 inches. In May, it was 3.4 inches. How much rain fell during the two months?

Estimate: $n > 8;\ n < 9$ n will be between 8 and 9.

Have children compute, then record the sum. They should describe the method they used.

$$\begin{array}{rl} 5\frac{3}{10} = 5 + \frac{3}{10} & \qquad 5\frac{3}{10} \\ \underline{3\frac{4}{10} = 3 + \frac{4}{10}} & \qquad \underline{3\frac{4}{10}} \\ 8 + \frac{7}{10} \text{ or } 8\frac{7}{10} & \qquad 8\frac{7}{10} \end{array}$$

$$\begin{array}{rl} 5.3 = 5 + .3 & \qquad 5.3 \\ \underline{3.4 = 3 + .4} & \qquad \underline{3.4} \\ 8 + .7 = 8.7 & \qquad 8.7 \end{array}$$

Have children compare the sum with the estimate.
Compare the various algorithms used and discuss the most efficient.

2. Subtraction: No Exchange

Suggested Problem: John walked 1.6 miles. Tom walked .5 of a mile. How much farther did John walk than did Tom?
$1.6 - .5 = n$

Estimate: About a mile.

Children compute then record the difference. They describe the method used. Encourage a variety of methods.

$$\begin{array}{rr} 1 + 6 \text{ tenths} & 1\frac{6}{10} \\ - \quad 5 \text{ tenths} & \frac{5}{10} \\ \hline 1 + 1 \text{ tenth} & 1\frac{1}{10} \end{array}$$

$$\begin{array}{rr} 1 + .6 & 1.6 \\ .5 & - \ .5 \\ \hline 1 + .1 & 1.1 \end{array} \qquad 1.6 - 0.5 = n$$

Have children compare the remainder with the estimate. Compare the algorithms and select the most efficient.

3. Provide practice:

a. Estimate the sum, then add. Compare the exact sum with the estimate. Verify by using the common fraction form.

$$\begin{array}{r} .2 \\ .4 \\ .3 \\ \hline \end{array} \qquad 0.5 + 0.1 + 0.3 \qquad \begin{array}{r} 5.2 \\ 7.4 \\ \hline \end{array} \qquad \begin{array}{r} 8.3 \\ 2.5 \\ 4.1 \\ \hline \end{array} \qquad \begin{array}{r} 37.2 \\ 14.6 \\ \hline \end{array} \qquad \begin{array}{r} 35.6 \\ 12.2 \\ 13.1 \\ \hline \end{array}$$

Find the sum of 54.2 and 37.6

Add: 14.7, 23.0, 19.2

b. Compare each of the following pairs of numbers and find the difference:

8.3, 5.0

6.7, 4.3

0.4, 5.6

19.5, 0.3

2.6, 17.9

27.3, 162.5

c. Solve the following:

$n + 6.4 = 23.7$

$29.8 + n = 35.9$

$238.6 - 129.3 = n$

From 138.9 take 59.7

Find the difference between 65.9 and 39.6

185.8 minus 28.6

Subtract 8.0 from 53.6

d. Without doing the computation, tell whether the solution of each of the following will be a whole number, a fraction, or a whole number and fraction.

0.7 + 0.4	0.8 + 0.6
.9 + .1	34.5 - 26.5

4. Additional exercises may be found in textbooks.

SETS; NUMBERS; NUMERATION

UNIT 61 - EXTENDED UNDERSTANDING OF SETS: INTERSECTION OF SETS

NOTE TO TEACHER

The intersection of two sets is the set which contains those elements and only those elements that are common to both sets. If A = {a, b, c} and B = {b, c, e } then the intersection of sets A and B is the set whose elements are b and c; {b, c }.

The symbol for intersection is ∩. The set which is the intersection of Set A and Set B is symbolized as A ∩ B. Here A ∩ B = {b, c}.

One of the applications of intersecting sets is in finding common denominators, common factors, etc.

Objectives: To reinforce concept of sets.
To develop meaning of intersection of sets.

TEACHING SUGGESTIONS

Reinforce Understanding of Sets, Set Notation, Set Union

1. Give an example of a set, then choose a subset of that set. For example, for the set of boys in our class, a subset may be {John, Fred}.

2. State the example given in question 1 above, in set notation. [{boys in our class} {John, Fred}]

3. $\{3, 4\} \cup \{\ \} = \square$ [{3, 4}]

4. $N\{3, 4\} + N\{\ \} = \square$ [2]

5. Are {a, b, c} and {d, e, f} equal sets? equivalent sets? Explain.

6. For $S = \{0, 1, 2, 3, 4, 5\}$

   Write the subset of S such that each number in the subset is even; is odd.

   Write the subset of S such that each element is a factor of 12; of 49.

Intersection of Sets

1. Suggested problem:

   John, Mary, Tom, Ralph and Sue belong to the Science Club.
   John, Ellen, Sue, Alice, David belong to the Math Club.
   Which children are members of both clubs? [John, Sue]

   Discuss set notation for this problem.

   A.

   The set of children in the Science Club = {John, Mary, Tom, Ralph, Sue}
   The set of children in the Math. Club = {John, Ellen, Sue, Alice, David}
   The set of children who are members of both clubs = {John, Sue}
   The intersection of the two sets = {John, Sue}

   B.

   If S is the set of members of Science Club and M is the set of members of Math. Club, then the intersection of S and M = {John, Sue}

2. Tell children that the symbol for intersection is $\cap$.

   For example:

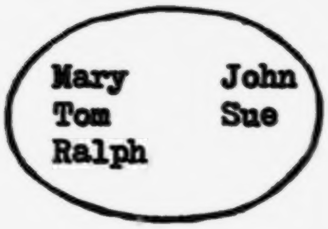
   {John, Mary, Tom, Ralph, Sue} ∩ {John, Ellen, Sue, Alice, David} = {John, Sue}

   or

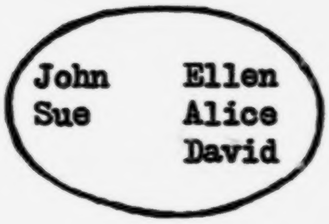
   $S \cap M = \{\text{John, Sue}\}$

3. Using diagrams to show intersection (Venn Diagrams) present and discuss the following:

Members of Science Club


Mary
Tom
Ralph
John
Sue

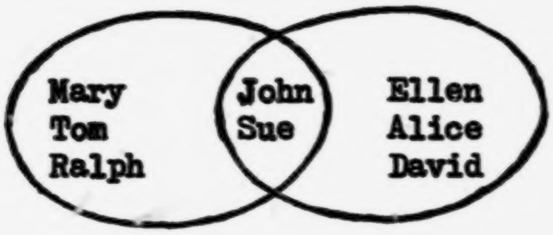
Members of Math Club


John
Sue
Ellen
Alice
David

John and Sue; Members of Both Clubs

Intersection of Science and Math Clubs

$S \cap M$


Mary
Tom
Ralph
John
Sue
Ellen
Alice
David

4. Provide practice

   a. Find the intersection of the following sets.
      Show in set notation.
      Then show using Venn Diagrams.

      {✱, ○, △, □, } and {◇, △, □, ▽, ⏢} [{△, □}]

      {a,b,c,d } and {e,b,g,h,d,k } [{b, d}]

      {0,1,2,3,4,5,6,7,8,9,10} and {1,3,5,7,9} [{1,3,5,7,9}]

   b. Which states border on New York?
      Which states border on Pennsylvania?
      Which states border on both New York and Pennsylvania?

      Show this as the intersection of two sets.

SETS; NUMBER; NUMERATION

UNIT 62 - SET OF FRACTIONAL NUMBERS: FINDING GREATEST COMMON FACTOR OF TWO NUMBERS; COMMON DENOMINATOR; LEAST COMMON DENOMINATOR

Objective: To help children apply the Fundamental Principle of Fractions. ($\frac{a}{b} = \frac{a \times c}{b \times c}$; $\frac{a}{b} = \frac{a \div c}{b \div c}$ where $c \neq 0$, $b \neq 0$) to find:

The greatest common factor of two numbers.
The common denominator for two or more fractions.
The least common denominator of two or more fractions.

TEACHING SUGGESTIONS

Finding the Greatest Common Factor

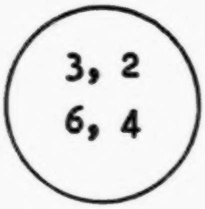
1. Reinforce renaming a fraction in simpler fractional forms.

For $\frac{8}{16}$: $\frac{8}{16} = \frac{\square}{8}$, $\frac{8}{16} = \frac{2}{\square}$, $\frac{8}{16} = \frac{\square}{2}$

What was the common factor by which you divided the numerator and the denominator in each case?

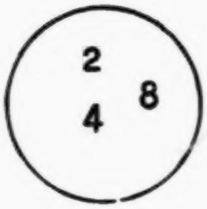
2. Use diagrams to show the intersection of the sets of factors. For example: For $\frac{12}{16}$

Factors of the Numerator



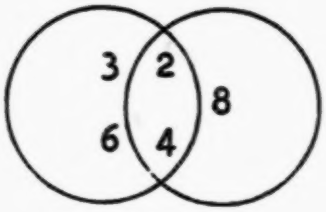
A

Factors of the Denominator



B

Factors Common to Numerator and Denominator



Children should note that:

$A = \{2, 3, 4, 6\}$

$B = \{2, 4, 8\}$

$A \quad B = \{2, 4\}$

Ask children:

What are the factors common to both?	[2, 4]
What is the largest factor common to both?	[4]

Direct children to change $\frac{12}{16}$ to simplest form.

Have children find an equivalent of $\frac{12}{16}$ using the greatest common factor as a divisor.

Children record the computation. $\left[\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}\right]$

What was the (greatest) common factor by which the numerator and the denominator were divided? [4]

What is the advantage of dividing the numerator and the denominator by the greatest common factor?

Finding A Common Denominator

1. Suggested problem: Which represents a larger number: $\frac{1}{2}$ or $\frac{7}{10}$?

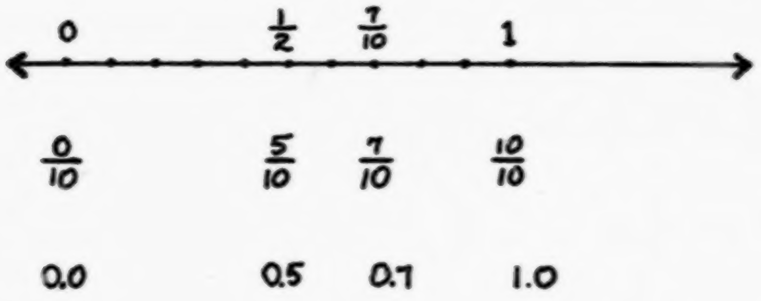
Have children draw a number line and label points corresponding to 0, $\frac{1}{2}$, $\frac{7}{10}$, 1.

They then compare the relative positions of $\frac{1}{2}$ and $\frac{7}{10}$.

Since $\frac{1}{2} = \frac{5}{10}$

and $\frac{7}{10} > \frac{5}{10}$

$\frac{7}{10} > \frac{1}{2}$



Follow the same procedure to compare $\frac{1}{4}$ with $\frac{3}{8}$, $\frac{1}{3}$ with $\frac{4}{9}$, $\frac{1}{2}$ with $\frac{5}{6}$, etc.

Discuss how the comparison was made in each case.

Tell children that: When two or more fractions have the same denominator they are said to have a common denominator.

In each set of fractions below have children select the fraction whose denominator can be used as a common denominator.

$\frac{1}{3}, \frac{1}{9}$ $\quad$ $\frac{1}{6}, \frac{1}{12}$ $\quad$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ $\quad$ $\frac{1}{10}, \frac{1}{50}, \frac{1}{100}$

2. Problem: To find a common denominator for $\frac{1}{2}$ and $\frac{1}{3}$.

Children rename $\frac{1}{2}$ and $\frac{1}{3}$

$$\frac{1}{2} = \frac{2}{\square}, \frac{3}{\square}, \frac{4}{\square}, \frac{5}{\square}$$

$$\frac{1}{3} = \frac{2}{\square}, \frac{3}{\square}$$

Teacher should record on chalkboard:

$$\begin{tikzcd}[column sep=small] \frac{1}{2} = & \frac{2}{4}, & \frac{3}{6}, & \frac{4}{8}, & \frac{5}{10} \\ \frac{1}{3} = & \frac{2}{6} \arrow[ur, leftrightarrow], & \frac{3}{9} & & \end{tikzcd}$$

Children should note that:

$\frac{1}{2}$ and $\frac{1}{3}$ can both be renamed as sixths, a common denominator.

The common denominator, 6, is not the denominator of either fraction.

3. Follow the same procedure to find a common denominator for:

$\frac{1}{3}$ and $\frac{1}{4}$; $\frac{1}{2}$ and $\frac{1}{5}$; $\frac{1}{3}$ and $\frac{2}{5}$

Ask children:

For the denominators 2 and 3, what was a common denominator? [6 or 12 or . . .]
For the denominators 3 and 4? [12]
for 3 and 5? [15]
Do you see a pattern?
How can you find common denominators without writing a set of equivalents for each fraction? [Generalization: The product of the denominators is always a common denominator.]

Finding The Least Common Denominator

1. **Reinforce** the concept that when adding or subtracting fractions with unlike denominators, children must rename the fraction so that the denominators are the same.

2. Suggested exercise: $\frac{1}{4} + \frac{1}{6} = n$

   Children should use the method of multiplying one denominator by the other to find a common denominator.

$$\begin{aligned} \frac{1}{4} + \frac{1}{6} &= \frac{6}{24} + \frac{4}{24} \\ &= \frac{10}{24} \\ &= \frac{5}{12} \end{aligned}$$

3. Discuss finding other denominators common to 4 and 6.

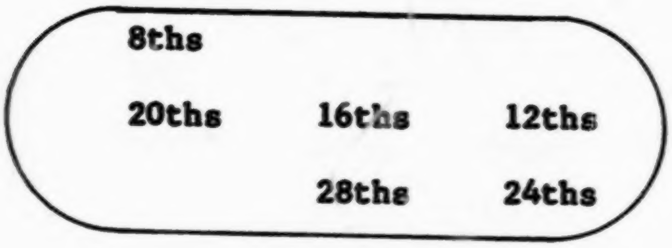
   Children rename $\frac{1}{4}$ in many ways; $\frac{1}{6}$ in many ways.

   Teacher records as shown below:

$$\begin{tikzcd}[column sep=small] \frac{1}{4} = & \frac{2}{8}, & \frac{3}{12}, & \frac{4}{16}, & \frac{5}{20}, & \frac{6}{24}, & \frac{7}{28}, & \frac{8}{32}, & \frac{9}{36} & \text{etc.} \\ \frac{1}{6} = & \frac{2}{12}, \arrow[ur, leftrightarrow] & \frac{3}{18}, & \frac{4}{24}, \arrow[urr, leftrightarrow] & \frac{5}{30}, & \frac{6}{36}, \arrow[urrr, leftrightarrow] & \text{etc.} & & & \end{tikzcd}$$

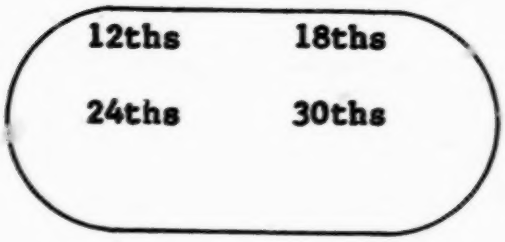
   Children note that 12, 24, and 36 are common denominators for 4 and 6, and that 12 is the Least Common Denominator.

Fourths may be renamed as


8ths
20ths
16ths
12ths
28ths
24ths

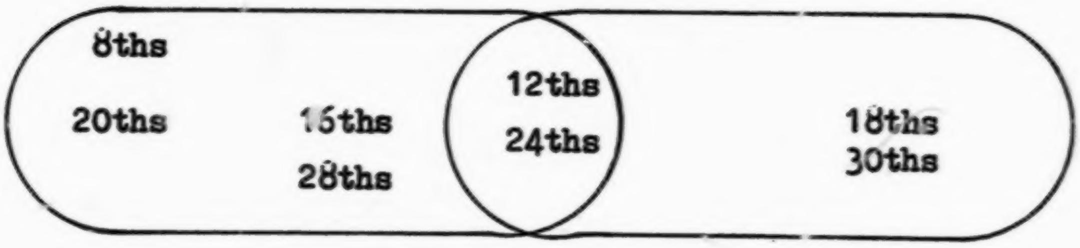
A

Sixths may be renamed as


12ths
18ths
24ths
30ths

B

Common Denominators for
Fourths and Sixths may be


8ths
20ths
16ths
28ths
12ths
24ths
18ths
30ths

Children should note that

$A = \{8, 12, 16, 20, 24, 28\}$

$B = \{12, 18, 24, 30\}$

$A \cap B = \{12, 24\}$

Have children note that common denominators may be found through the intersection of sets of multiples of the two denominators.

4. Have children solve the exercise, $\frac{1}{4} + \frac{1}{6} = n$, using 12 as the common denominator; 24 as the common denominator. They compare the computation and discover the advantage of using the least common denominator.

5. Suggested exercise: $\frac{1}{6} + \frac{1}{8} = n$.

   Children find a common denominator for $\frac{1}{6}$ and $\frac{1}{8}$.

   Is it the least common denominator?

Discuss various ways of finding the least common denominator.

> Is the larger denominator (8) common to both?
> If not, is 2 times the larger denominator common to both? 3 times? etc.

Children state this method for finding the least common denominator in their own words. (Multiply the larger denominator by 2, 3, etc. until a common denominator is reached.)

*6. Some children may state the above in terms of multiples. (Optional)

Ask children to write the multiples of $\{8, 16, 24, 32, \ldots\}$

> Which is the smallest multiple of 8 that is also a multiple of 6?

Generalization: The least common multiple (24) of the denominators (6 and 8) is the least common denominator.

Extend to finding the least common denominator for 3 or more fractions.

EVALUATION AND / OR PRACTICE

SUGGESTED EXERCISES

1. Find the least common denominator for:

   $\frac{1}{3}$ and $\frac{1}{4}$ $\quad$ $\frac{1}{3}$ and $\frac{1}{10}$ $\quad$ $\frac{1}{8}$ and $\frac{1}{5}$ $\quad$ $\frac{1}{12}$ and $\frac{1}{9}$

2. Which fraction is larger? Why?

   $\frac{3}{4}$ or $\frac{5}{6}$ $\quad$ $\frac{1}{3}$ or $\frac{2}{5}$ $\quad$ $\frac{3}{10}$ or $\frac{1}{6}$ $\quad$ $\frac{1}{4}$ or $\frac{2}{9}$

3. Find the least common denominator for:

   $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$ $\quad$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$

4. Which fraction is the largest within each set? Why?

   $\frac{2}{3}, \frac{1}{2}, \frac{3}{5}$ $\quad$ $\frac{5}{5}, \frac{6}{7}, \frac{2}{3}$ $\quad$ $\frac{3}{4}, \frac{5}{8}, \frac{2}{3}$

5. Find common denominators for $\frac{1}{8}$ and $\frac{1}{6}$, using the diagrams for intersection of sets.

6. Additional practice exercises may be found in textbooks.

OPERATIONS

UNIT 63 - SET OF FRACTIONAL NUMBERS: ADDITION AND SUBTRACTION; COMMON FORM; HORIZONTAL AND VERTICAL FORMAT

Objective: To help children add and subtract fractions, common form, using the least common denominator method.

TEACHING SUGGESTIONS

1. Reinforce counting forward and backward.

   Study each sequence below. Find the pattern that was used, then fill in the blanks.

   a. $1 \quad 1\frac{1}{4} \quad 1\frac{1}{2} \quad 1\frac{3}{4}$ - - - - -.

   b. $\frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3}$ - - - -.

   c. $1 \quad \frac{15}{16} \quad \frac{7}{8} \quad \frac{13}{16} \quad \frac{3}{4}$ - - - -.

   d. $\frac{1}{9} \quad \frac{1}{6} \quad \frac{2}{9}$ - - - - - -.

   e. $2\frac{1}{3} \quad 2\frac{1}{12} \quad 1\frac{5}{6} \quad 1\frac{7}{12}$ - - -.

2. Reinforce renaming fractions.

   a. Change each of the following to an equivalent fraction in higher terms.

   $12\frac{3}{8}$: $\quad 12\frac{\square}{16}$, $\quad 12\frac{\square}{32}$

   $6\frac{2}{3}$: $\quad 6\frac{\square}{9}$, $\quad 6\frac{\square}{6}$, $\quad 6\frac{\square}{12}$

b. Change each of the following to equivalent fractions in lowest terms.

$12\frac{10}{20}$: $12\frac{2}{\square}$, $12\frac{1}{\square}$, $12\frac{5}{\square}$

$\frac{8}{16}$: $\frac{2}{\square}$, $\frac{1}{\square}$, $\frac{4}{\square}$

3. Reinforce regrouping.

Complete the following to make the statement true:

$14\frac{3}{8} = \square\frac{11}{8}$ $26\frac{3}{10} = 25\frac{13}{\square}$ $129\frac{3}{5} = 128\frac{\square}{5}$

__Addition and Subtraction: Horizontal Format__

1. Reinforce adding and subtracting fractions with unlike denominators - common denominator apparent.

Suggestions for evaluation:

Find the missing number, then solve the equation.

$\frac{11}{16} + \frac{9}{16} = 1 + \frac{\square}{4}$

$15\frac{5}{8} - 3\frac{1}{2} = 12\frac{5}{8} - n$

$5\frac{1}{2} + \frac{7}{10} = 6 + \frac{\square}{5}$

$7\frac{1}{5} - 1\frac{4}{5} = 6\frac{1}{5} - n$

$17\frac{5}{6} + 8\frac{5}{12} = 25 + \frac{\Delta}{\square}$

$10\frac{1}{3} - \frac{7}{9} = 9 - n$

$24\frac{7}{18} + 48\frac{2}{3} = n$

$35\frac{3}{14} - 18\frac{3}{7} = 16 - n$

2. **Suggested exercises: Fractions with unlike denominators.**

a. $5\frac{1}{2} + 2\frac{1}{5} = n$

Children may think:

$5\frac{1}{2},\ 7\frac{1}{2},\ 7\frac{5}{10},\ 7\frac{7}{10}$ or $5\frac{1}{2},\ 5\frac{5}{10},\ 7\frac{5}{10},\ 7\frac{7}{10}$

or

$$5\frac{1}{2} + 2\frac{1}{5} = (5 + 2) + (\frac{1}{2} + \frac{1}{5})$$

$$= 7 \quad + (\frac{5}{10} + \frac{7}{10}) = 7\frac{7}{10}$$

b. $12\frac{2}{3} - 8\frac{1}{2} = n$

Children may think:

$$12\frac{2}{3} - 8\frac{1}{2} = 4 + (\frac{2}{3} - \frac{1}{2})$$

$$= 4 + (\frac{4}{6} - \frac{3}{6}) = 4\frac{1}{6}$$

or

$12\frac{2}{3},\ 4\frac{2}{3},\ 4\frac{4}{6},\ 4\frac{1}{6}$

or

$12\frac{2}{3},\ 12\frac{4}{6},\ 4\frac{4}{6},\ 4\frac{1}{6}$

3. **Provide practice. Suggested Exercises:**

Have children find the missing number, then solve the equation.

$\frac{2}{3} + \frac{3}{4} = \frac{8}{\square} + \frac{9}{\square}$ $\qquad$ $\frac{1}{2} - \frac{1}{7} = \frac{\Delta}{14} - \frac{\square}{14}$

$5\frac{4}{9} + \frac{1}{2} = 5 + n$ $\qquad$ $8\frac{5}{7} - \frac{1}{3} = n$

$6\frac{2}{3} + \frac{5}{8} = 7 + n$ $\qquad$ $\frac{2}{3} - \frac{1}{2} = \frac{4}{\square} - \frac{3}{\square}$

$\frac{5}{6} + \frac{1}{4} = \frac{10}{\square} + \frac{3}{\square}$ $\qquad$ $16\frac{2}{3} - 7\frac{3}{5} = n$

Vertical Format

1. **Reinforce adding and subtracting fractions with unlike denominators: common denominator apparent.**

Suggested exercises:

a. **Find the sums:**

$$\begin{array}{r} 36\frac{4}{5} \\ 18\frac{3}{10} \\ \hline \end{array} \qquad \begin{array}{r} 175\frac{6}{8} \\ 89\frac{3}{16} \\ \hline \end{array}$$

b. **Find the remainders:**

$$\begin{array}{r} 59\frac{2}{3} \\ -\ 23\frac{3}{9} \\ \hline \end{array} \qquad \begin{array}{r} 61\frac{7}{12} \\ -\ 35\frac{5}{6} \\ \hline \end{array}$$

2. **Reinforce finding Least Common Denominator for:**

Eighths and tenths
Sixths and fourths
Sixths and ninths **etc.**

3. Develop addition and subtraction of fractions with unlike denominators: Least Common Denominator Method

Children should estimate sums and remainders before computing. They find a least common denominator before computing. Illustrative exercises:

a. $6\frac{4}{5} + 9\frac{2}{3} = n$

Estimate: $6\frac{4}{5} + 9 = 15\frac{4}{5}$, therefore $6\frac{4}{5} + 9\frac{2}{3} > 15\frac{4}{5}$

or

$6\frac{4}{5} > 6$, $9\frac{2}{3} > 9$, $6\frac{4}{5} < 7$, $9\frac{2}{3} < 10$; therefore

$6\frac{4}{5} + 9\frac{2}{3}$ must be greater than 15 and less than 17

Compute:

$$\begin{array}{rcll} 6\frac{4}{5} & = & 6\frac{12}{15} & \\ \underline{9\frac{2}{3}} & = & \underline{9\frac{10}{15}} & \\ & & 15\frac{22}{15} & = 16\frac{7}{15} \end{array}$$

Have children compare solution with estimate.

b. $45\frac{5}{6} - 27\frac{2}{9} = n$

Estimate: $45\frac{5}{6} - 27 = 18\frac{5}{6}$, therefore $45\frac{5}{6} - 27\frac{2}{9} < 18\frac{5}{6}$

Compute:

$$\begin{array}{rcl} 45\frac{5}{6} & = & 45\frac{15}{18} \\ \underline{-\ 27\frac{2}{9}} & = & \underline{27\frac{4}{18}} \\ & = & 18\frac{11}{18} \end{array}$$

Have children compare solution with estimate.

c. Subtraction exercise where regrouping is necessary for exchange: $57\frac{1}{4} - 39\frac{5}{6} = n$

Children should estimate before computing.

Compute:

$$\begin{array}{rcrcr} 57\frac{1}{4} & = & 57\frac{3}{12} & = & 56\frac{15}{12} \\ \underline{-\ 39\frac{5}{6}} & = & \underline{39\frac{10}{12}} & = & \underline{39\frac{10}{12}} \\ & & & & 17\frac{5}{12} \end{array}$$

Have children explain why $57\frac{3}{12}$ should be renamed as $56\frac{15}{12}$.

4. Answers should be verified. A suggested algorithm for verification follows:

Computation (Vertical Format)

Exercise: $17\frac{2}{3} + 25\frac{3}{4} = n$

$$\begin{array}{rcl} 17\frac{2}{3} & = & 17\frac{8}{12} \\ \underline{25\frac{3}{4}} & = & \underline{25\frac{9}{12}\qquad\quad} \\ & & 42\frac{17}{12} = 43\frac{5}{12} \end{array}$$

Verification (Horizontal Format)

$$\begin{aligned} 17\frac{2}{3} + 25\frac{3}{4} &= (17 + 25) + (\frac{2}{3} + \frac{3}{4}) \\ &= 42 + (\frac{8}{12} + \frac{9}{12}) \\ &= 42 + \frac{17}{12} \\ &= 43\frac{5}{12} \end{aligned}$$

5. Continue to provide practice in addition and subtraction of fractions using both horizontal and vertical formats. Additional suggestions may be found in textbooks.

6. Verbal problems may be chosen from situations involving geometric figures such as finding the perimeter of squares, rectangles. For example:

   a. The perimeter of a square is 7 in. Its width is $1\frac{3}{4}$ in. Find the length of the rectangle. $3\frac{1}{2} - 1\frac{3}{4} = \square$

   b. The base of an equilateral triangle is $1\frac{3}{4}$ in. long. Find its perimeter. (Solve through addition)

OPERATIONS

UNIT 64 - SET OF FRACTIONAL NUMBERS: ADDING AND SUBTRACTING TENTHS; DECIMAL FORM; WITH EXCHANGE

Objectives: To help children maintain skill in adding and subtracting decimal fractions mentally.

To develop vertical algorithms involving decimal fractions; with exchange.

TEACHING SUGGESTIONS

Horizontal Format

1. As children add "mentally" they may begin with the entire first addend. When they subtract "mentally" they may begin with the entire minuend.

For $3.4 + 5.8 = n$ children may think:

$$\begin{array}{lcl}
3.4 + 5.8 & & \\
= (3.4 + 5) + .8 & & 3.4 + 5.8 = n \\
= (8.4 + .6) + .2 & \text{or} & = (3 + 5) + (.4 + .8) \\
= \quad 9 \quad + .2 & & = \quad 8 \quad + \quad 1.2 \\
= 9.2 & & = 9.2
\end{array}$$

For $9.2 - 5.8 = n$ children may think:

$$\begin{array}{ll}
9.2 - 5.8 & \\
= (9.2 - 5) - .8 & \text{Regrouping } 5.8 \text{ as } 5 + .8 \\
= (4.2 - .2) - .6 & \text{Subtracting } 5 \text{ from } 9.2\text{;} \\
 & \text{regrouping } .8 \text{ as } .2 \text{ and } .6 \\
= \quad 4 \quad - .6 & \text{Subtracting } .2 \text{ from } 4.2 \\
= 3.4 &
\end{array}$$

Encourage other ways of arriving at solutions.

2. Suggested exercises:

Addition

$26.7 + 5.7 = n$ [26.7, 31.7, 32, 32.4]
$142.9 + 3.2 = n$ [142.9, 145.9, 146, 146.1]
$32.5 + 41.4 = n$
$16.9 + 12.2 = n$

Subtraction

$32.4 - 5.7 = n$ [32.4, 27.4, 27, 26.7]
$146.1 - 3.2 = n$ [146.1, 143.1, 143, 142.9]
$45.8 - 24.5 = n$
$29.1 - 12.2 = n$

Vertical Format

1. Refer to Unit 59 to reinforce adding and subtracting tenths without exchange.

2. Introduce addition with exchange.

Suggested problem: Tom rode 3.5 miles to Frank's house and then 2.7 miles to the library. How far did Tom travel?

$$3.5 + 2.7 = n$$

Estimate: $n > 5$; n is between 5 and 7.

Have children solve using fractional form first.

$$\begin{array}{l} 3\frac{5}{10} \\ 2\frac{7}{10} \\ \hline 5\frac{12}{10} = 6\frac{2}{10} \end{array}$$

Develop the algorithm for decimal form. Discuss as you proceed.

$$\begin{array}{rl} 3.5 &= 3 + 5 \text{ tenths} \\ \underline{2.7} &= \underline{2 + 7 \text{ tenths}} \quad \text{(Renaming)} \\ & \, 5 + 12 \text{ tenths (Sum)} \\ &= 5 + 1 + 2 \text{ tenths (Renaming 12 tenths)} \\ &= 6 + 2 \text{ tenths} \\ &= 6.2 \end{array}$$

As you discuss each step above, record in decimal form as shown below.

A	B
$3.5 = 3 + 5$ tenths	$3.5 = 3 + .5$
$\underline{2.7} = \underline{2 + 7 \text{ tenths}}$	$\underline{2.7} = \underline{2 + .7}$
$5 + 12$ tenths	$5 + 1.2$
$= 6 + 2$ tenths	$= 6.2$

Solve the problem again, using the concise form.

$$\begin{array}{r} 3.5 \\ \underline{2.7} \\ 6.2 \end{array}$$

Compare solution with the estimate.

3. Introduce subtraction with exchange.

   a. Reinforce renaming the minuend.

      $6.3 = 5 + 13$ tenths
      $7.5 = 6 + 15$ tenths
      $9.2 = 8 + 12$ tenths
      $4.7 = 3 + \square$ tenths
      $5.2 = \square + 12$ tenths
      $3.8 = 2 + \square$ tenths

b. Suggested problem: The distance from Tom's house to the park is 6.2 miles. His cousin's house is 2.9 miles from the park. How much farther away is Tom's house?
$6.2 - 2.9 = n$

Estimate: About 3 miles; $n < 4$; between 3 and 4 miles

Have children solve the problem, using fractional form first.

$$\begin{array}{rcl} 6\frac{2}{10} & = & 5\frac{12}{10} \quad \text{(Renaming)} \\ -\ \underline{2\frac{9}{10}} & = & \underline{2\frac{9}{10}} \\ & & 3\frac{3}{10} \end{array}$$

Then solve the same exercise using decimal form. Discuss each step.

$$\begin{array}{lcr} \text{A} & & \text{B} \\ 6.2 = 6 + .2 = 5 + 1.2 & & 6.2 \\ -\underline{2.9} = \underline{2 + .9} = \underline{2 + \ \ .9} & & -\ \underline{2.9} \\ \qquad\qquad\qquad\quad 3 + \ .3 = 3.3 & & 3.3 \end{array}$$

Compare the answer with the estimate.

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

1. Find the sum of: 9.5, 2.7, 23.9
   Write in a column and add: 25.8, 67.0, 48.6, 0.3
   Find the missing numeral. $628.9 + 23.5 = n$

2. Verify sums by converting to fractional form.

3. Verify remainders by using the fractional form or by adding:

   From 20.0 subtract 7.6
   From 124.7 subtract 58.9
   128.5 minus 73.9
   Subtract 24.7 from 132.2
   Find the difference between 135.8 and 352.3
   Find the missing numeral:
   $53.2 - n = 34.6$
   $20.4 + n = 100.2$
   $n + 236.8 = 325.0$

4. In the following pairs of numbers, subtract the smaller from the larger.

   36.3, 18.8 　　60.4, 307.2
   2.4, 21.3 　　4.2, 0.5

5. Additional exercises may be found in textbooks.

GEOMETRY AND MEASUREMENT

UNIT 65 - GEOMETRY: CIRCLE

Objectives: To extend understanding of simple closed curves in a plane to include circles.

To develop the definition of a circle as a simple closed plane curve formed by the set of all points which are the same distance from a fixed point in the plane called the center.

To extend understanding of geometric terms to include: circumference, radius, diameter, chord, arc.

TEACHING SUGGESTIONS

Characteristics of a Circle

1. Reinforce meaning of simple closed curve; meaning of plane; meaning of polygon.

2. Circumference

   Have children mark a point on a paper. Label it C. Mark several points, all 1 inch from C.

   Ask children:

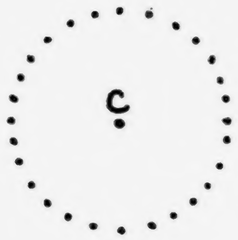
   How many points can you mark 1 inch from C?

   What figure is being formed by all of these points?

   Is a circle a simple closed figure? Explain.

   How does a circle differ from a polygon?

   How would you describe point C in relation to the points of the circle?



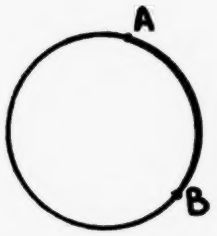
Tell children that:

A circle is a simple closed curve on a plane, formed by the set of all points which are the same distance from a fixed point in the plane called the center.

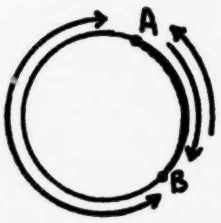
Circumference is the distance around the circle or the length of the circle.

3. Arc

Have children use a plastic compass to draw a circle.
Label a point on the circle A.
Name another point on the circle B.



Trace the two paths on the circle from A to B;
from B to A.



Tell children that the part of a circle between any two points is called an arc of the circle.
Discuss the length of the arc in relation to the circumference of the circle.

4. Radius

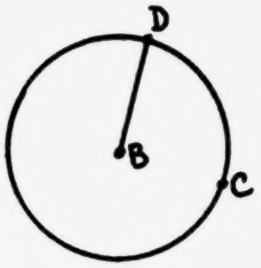
Children:

Draw a circle.

Label the center point B.

Label point D on the circle.

Draw line segment $\overline{BD}$.



Tell children $\overline{BD}$ is called a radius of the circle. (Here "radius" refers to the line segment. It can also be used to refer to the length of the segment.)

Have children:

Label another point C on the circle.
Draw $\overline{BC}$.

Discuss:

Is $\overline{BC}$ a radius of the circle? Why?

Can you draw another radius?

How many radii does a circle have?

Are all the radii the same length?

How can you find out?

Children describe a radius as:

A line segment from the center to any point of the circle. The length of any radius is also referred to as the "radius" of the circle.

5. Chord

Have children:

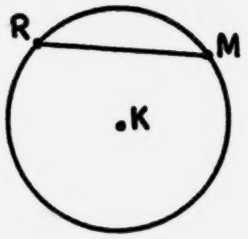
Draw a circle with a plastic compass.

Label its center K.

Mark two points on the circle.

Label them R and M.

Draw the line segment $\overline{RM}$



Have children:

Choose any other two points on the circle.

Label them.

Draw a line segment with those points as endpoints.

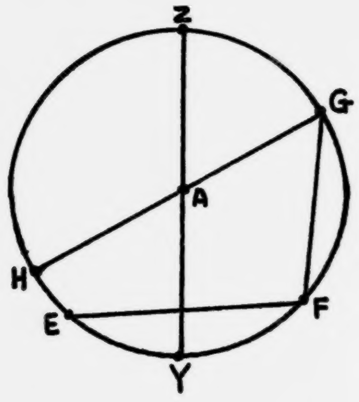
Tell children that any line segment that has its two endpoints on the circle is called a chord.

Is a radius a chord?

Is there a largest chord?

6. Diameter

Children examine the diagram of the circle whose center is A.



Name some of the chords.

What is the difference between chords $\overline{GF}$ and $\overline{GH}$?;

between $\overline{EF}$ and $\overline{ZY}$?

[$\overline{GH}$ and $\overline{ZY}$ have endpoints on the circle and pass through the point at the center.]

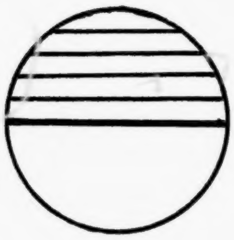
Tell children that:

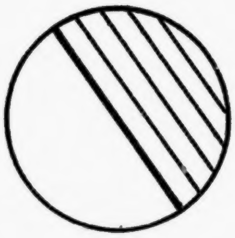
A chord that passes through the center is called the diameter.

The length of any diameter is also referred to as the diameter of the circle.

Have children:

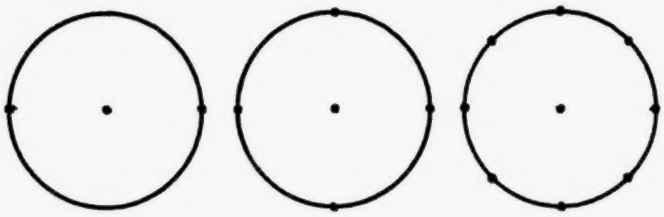
Draw a circle. Draw several diameters.
Discover that the longest chord passes through the center.
Diameters are the longest chords of a circle.



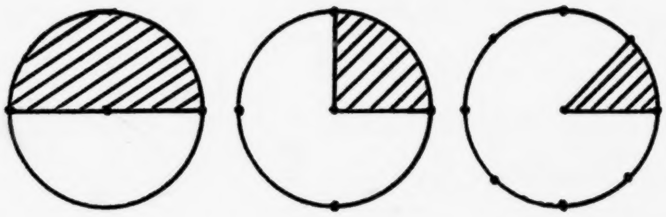


7. Central Angle

a. Provide a sheet of rexographed circles.
Have children divide successive circles in halves, fourths, eighths, thirds, sixths, etc.



Draw and shade one central angle in each as shown:

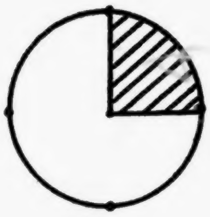


Discuss the meaning of central angle as the angle formed by 2 radii. (Radii is the plural of radius). Since two radii are involved, the vertex is at the center of the circle.

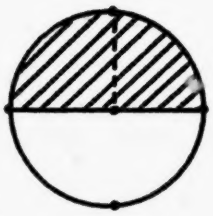
b. Describe and compare the central angles:

The arc that has the measure of:

$\frac{1}{4}$ of the circle subtends a right angle.

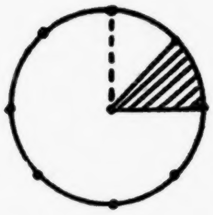


$\frac{1}{2}$ of the circle subtends a straight angle. (twice the size of a right angle)

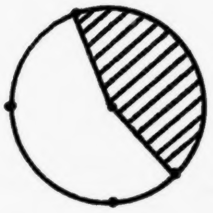


$\frac{1}{2}$ of a circle is called a semicircle

$\frac{1}{8}$ of the circle subtends an acute angle. (smaller than a right angle)



$\frac{1}{3}$ of the circle subtends an obtuse angle. (greater than a right angle and less than a straight angle)



Discuss central angles subtended by $\frac{1}{16}$ of a circle; $\frac{1}{5}$ of a circle; $\frac{3}{8}$ of a circle; $\frac{5}{8}$ of a circle, etc.

Discuss central angles formed by the two hands of a clock:

2 P.M. (acute)
3 P.M. (right)
4 P.M. (obtuse) etc.

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

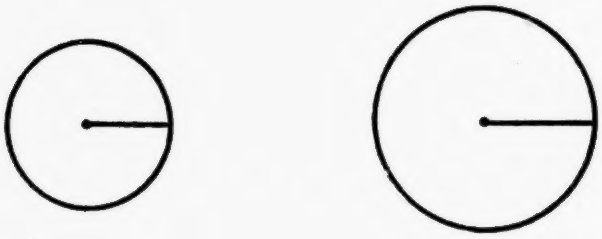
1. Draw a diagram to show whether the sentences below are true or false.

    a. A circle has all its points the same distance from a point inside called the center.

    b. All the radii of a circle have the same length.

2. Draw two different circles so that the radius of one has the same length as the radius of the other.

    Draw two different circles so that one has a radius of a different length from the other.



    Measure the length of each radius.

    Extend each radius through the center to form a diameter.

    Measure the length of the diameters.

    Determine the relationship between a diameter and a radius of each circle.

3. Draw a circle. Draw chords to form a square within the circle; a rectangle within the circle. Draw the diagonals of the square. What kind of central angles do they form?

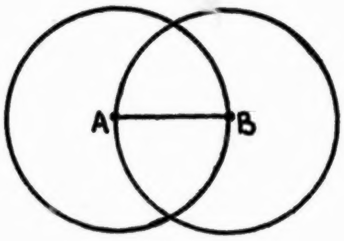
    Draw the diagonals of the rectangle. What kind of central angles do they form?

    What is true of the chords that form the square?

    [They are all the same length]

4. If the radius of a circle is 6 inches long, what is the length of the diameter?

5. Draw a line segment $\overline{AB}$. Then draw the two circles which have $\overline{AB}$ as a radius.



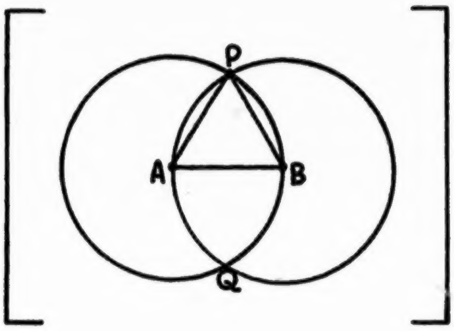
Ask children:

What is the center of one circle? [A]

Of the other circle? [B]

At how many points do these circles intersect? [2]

Label these points P and Q. Draw $\overline{AP}$ and $\overline{BP}$.



Use your compass or a ruler to determine:

Is $\overline{AP}$ equal to $\overline{AB}$?

Is $\overline{BP}$ equal to $\overline{AB}$?

Then, is $\overline{AP}$ equal to $\overline{BP}$?

What kind of triangle is Δ APB? [Equilateral]

GEOMETRY AND MEASUREMENT

UNIT 66 - MEASUREMENT: GRAPHIC REPRESENTATION; BAR GRAPH

NOTE TO TEACHER

A Graph pictures a type of numerical relationship between 2 kinds of information.

There are several kinds of graphs: Pictographs, Bar Graphs, Line Graphs, Graphs of Solution Sets of Open Sentences, etc.

A pictograph pictures information by means of pictures; bar graphs show comparisons by means of bars; line graphs show trends or changes in data by means of lines.

Children should be taught to interpret graphs before they are taught to construct graphs.

Objectives: To teach children the meaning of Scale and Axis.

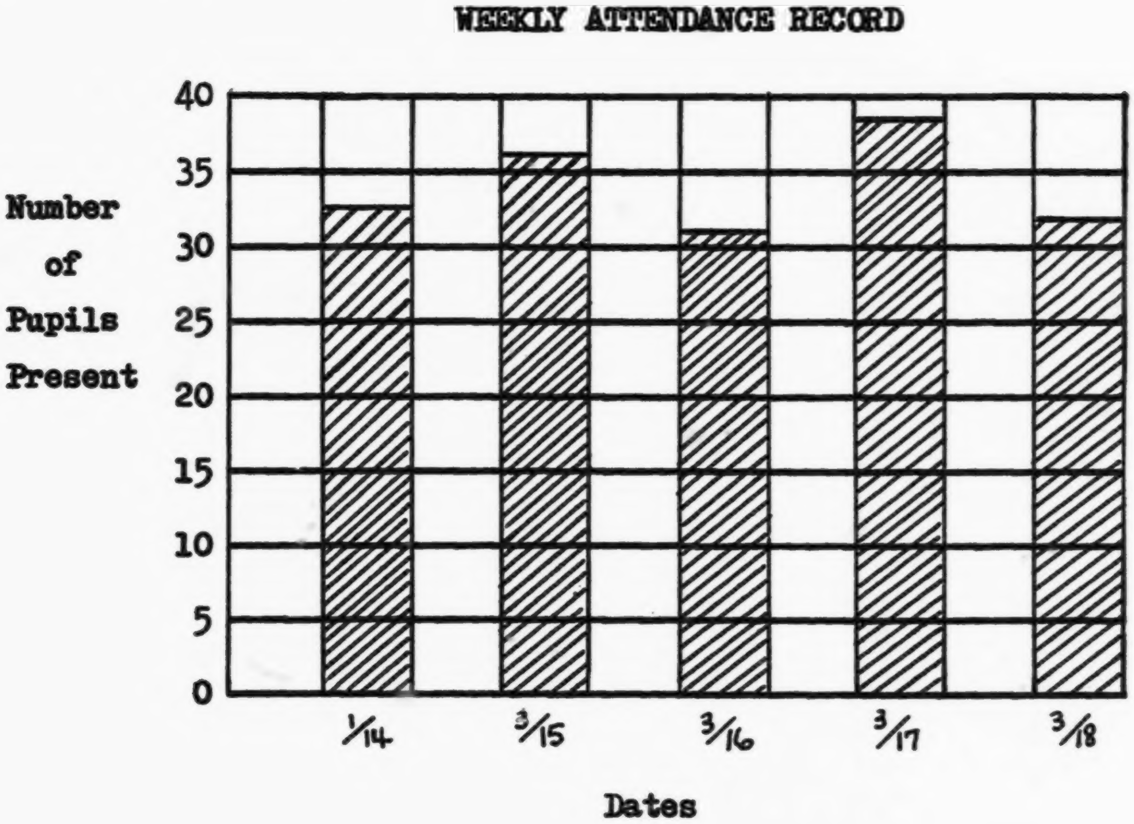
To help children:

- Read and interpret bar graphs.
- Construct bar graphs.

TEACHING SUGGESTIONS

Reading and Interpreting Bar Graphs

1. Present a graph such as the following:



Discuss:

What does this graph show?

What is the title of the graph?

What do the dates at the bottom indicate?

What do the numbers at the left show?

On what date was the attendance highest?

About how many children were present on March 17?

On what date were the most children absent?

How many more children were absent on March 16 than on March 15?

On what days was the attendance perfect?

On what days was the attendance between 35 and 40?

How could we obtain the data for this graph?

Tell children that each bar graph has a scale.

Discuss:

Meaning of scale. [Scale is a measure shown by markings at regular intervals.]

The scale on the graph above. What are the intervals?

Where do they begin? [0]

Scale may be shown on horizontal or vertical line segment.

Tell children that the vertical line segment and the horizontal line segment are each called an axis. (Plural is axes).

2. Ask children to bring bar graphs out from magazines or newspapers.

Have children study their bar graphs and note:

Bar Graphs usually have a scale which starts at zero.

Graphs should have a title, and a label for each axis.

Bars should be the same width.

Depending upon the interval selected, the numbers of the data may have to be shown approximately to the nearest 10, 100, 1000, etc.

The scale is marked on one of the axes.

Each bar should be labeled.

3. Have children compare a bar graph with a chart to determine the advantages of each way of showing records or other numerical information.

WEEKLY ATTENDANCE RECORD

Dates	No. of Pupils Present	
Mar. 14	34	
15	36	
16	32	
17	38	
18	36	(data same as bar graph)

Children should understand that:

BAR GRAPHS are used to facilitate making general comparisons. They are usually not meant to convey information with any high degree of precision. This can be shown much more accurately by charts that list data.

Constructing Bar Graphs

Bar Graphs can be used to show comparisons of:

Temperature readings	Incomes
Attendance records	Populations, etc.

The necessary data may be found in charts or tables, or may be gathered by the children.

1. Suggested Problem: On Tuesday, 432 people came to our dance festival, 549 came on Wednesday and 324 on Thursday. Show this on a chart, on a horizontal bar graph, on a vertical bar graph.

Table

Attendance at Dance Festival	
Day	Number of People
Tues.	432
Wed.	549
Thurs.	324

Questions to ask children as they construct graph:

What shall we label each axis?

What scale shall we use?

How many spaces will we need?

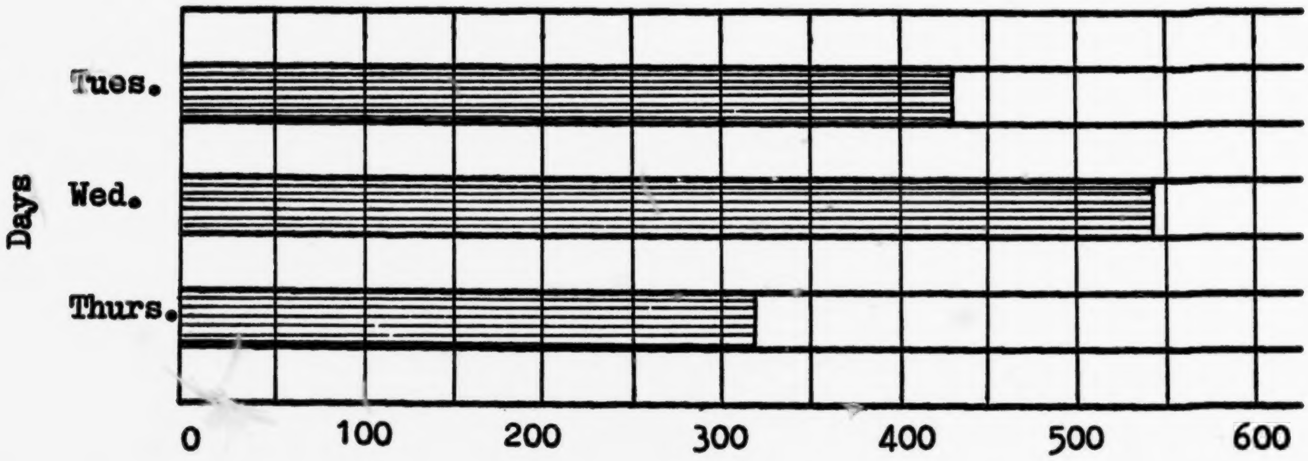
How shall we mark the intersection of the vertical and horizontal axes?

How can we plan so that the bars are the same width and the spaces are the same distance apart?

How shall we decide upon the size of the graph?

Horizontal Bar Graph

Attendance at Dance Festival

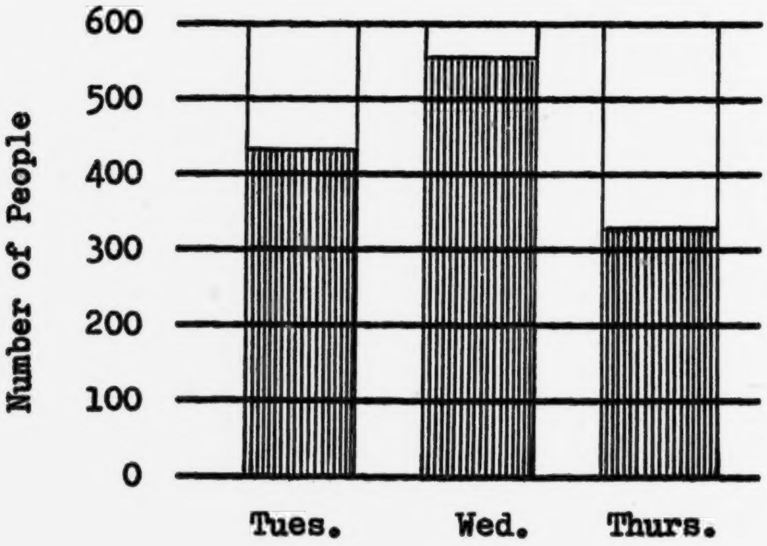


Number of People

Note the placement of the numerals at the intersection of vertical and horizontal lines.

Vertical Bar Graph

Attendance at Dance Festival



2. Suggested Practice:

Have children construct bar graphs based on experiences and data found in mathematics textbooks, newspapers and social studies textbooks.

Compare bar graphs with the tables and discuss the value of both ways of showing sets of data.

Which is of greater value pictorially? numerically?

Questions may be based on information shown in the graph.

On what day was the attendance highest? lowest? etc.

OPERATIONS

UNIT 67 - STATISTICS: FINDING AVERAGES

NOTE TO TEACHER

An average is one number that gives information about a set of data. An average is a balance point.

Although children often use the word "average," the concept is generally not understood. Finding an average means a reorganization of unequal groups to arrive at an equal although imaginary, distribution. An average can be obtained by a redistribution process. Finding averages is a use of partitioning division.

Objectives: To help children understand the meaning of average and its use in the interpretations of data.

To develop the procedure for finding the average of a set of numbers.

TEACHING SUGGESTIONS

Finding an Average When the Average is One of the Numbers in the Data

Using Objects

1. Suggested problem: Three committees brought in reference books for research in Social Studies. Committee A brought 11 books, Committee B, 5 books and Committee C, 8 books. If each committee is to receive the same number of books, how many would be in each group?

2. Have children arrange books brought by each committee and record the number.

A	B	C
11 books	5 books	8 books

3. Children should estimate the number each committee is to receive before attempting to find the answer.

   Discuss:

   Will each committee receive as many as 11 books?

   As few as 5? Why not?

   Somewhere between 5 and 11? Where?

4. Have children "even up" the stacks of books.
   Compare the number in the re-distributed groups with the original number of books in each group.

   Children should note that there are 8 books now in each stack.

   They compare 11 with 8. It is 3 more.

   They compare 5 with 8. It is 3 less.

   They compare 8 with 8. It is the same.

   They should see that there were 3 more books in one stack and 3 less in the other stack.

   8 is the "in-between" number.

   8 is the balance point or a "midpoint."

5. Tell children that the number of books in the redistributed books is the mid-value or an average.

6. Tell children that there is a faster way to find an average.

   a. Have children arrange the books as they were when each committee brought them in $\begin{matrix} A & B & C \\ 11 & 5 & 8 \end{matrix}$.

      Then have children pile the books into a single stack.

      As they stack they add the number of books. [11 + 5 + 8]

      Help children, through questioning, to discover that stacking the books into one group and then dividing them into 3 equal parts will give the average.

On the chalkboard record:

$$\begin{array}{r}11\\5\\\underline{\;8}\\24\end{array}\qquad\qquad \begin{array}{r}\underline{8\text{ books}}\\3\,\overline{)24\text{ books}}\end{array}$$

8 books is the average number of books each committee would have brought in if each had brought the same number of books.

b. Suppose 4 committees brought in reference books. Committee A brought in 11 books, Committee B, 5 books, Committee C, 8 books and Committee D 12 books. If each committee is to receive the same number of books, how many would each receive?

Have children follow the same procedure as for items 2 - 6 above.

Children discuss after the recording:

$$\begin{array}{r}11\\5\\8\\\underline{12}\\36\end{array}\qquad\qquad \begin{array}{r}\underline{9\text{ books}}\\4\,\overline{)36\text{ books}}\end{array}$$

that 9 books is the average number of books each committee would have brought in if each had brought the same number of books.

Help children, through questioning, to see that stacking the books into one group and then dividing them into 4 equal parts will give the average.

Children state the generalization for the method of finding an average as it applies to the situations above. For example, add and then divide by the number of piles.

Using Numbers Only

1. Suggested problem: Sally's arithmetic test marks were 70, 80, 75, 70, 80. What was her average arithmetic test mark?

Suggest that children arrange the numbers in order and use the middle value as a first estimate.
For: 70, 70, 75, 80, 80
Estimate: 75

2. Children compare each number with the estimate, 75.
   They note:

   70 is 5 below the middle value

   70 is 5 below the middle value

   75

   80 - - - - - - - - - - - - - - - is 5 above the middle value

   80 - - - - - - - - - - - - - - - is 5 above the middle value

   10 below — 10 above

   Solution: 75, since the amount below (10) balances the amount above (10). Find the solution by adding the marks and dividing the sum into 5 equal parts.

3. Children should state the generalization for the method of finding an average.

4. Find the average of each of the following sets of numbers and check your solution.

   17, 7, 12 39, 15, 27 8, 12, 10, 4, 6

   56, 62, 68, 59, 65

Average Is Not One of the Numbers in the Data

1. Suggested problem: Find the average temperature for the school day if our recordings show that it was 68° in the morning, 74° at noon and 77° in the afternoon.

2. Children try the midvalue at 74° and observe that it is not the average.

   Children note:

   68° is 6° below 74°

   74°

   77° - - - - - - - - is 3° above 74°

   6° below — 3° above

Children should note that:

6° and 3° do not balance.
Their estimate of 74° was too large because there are more degrees below it than above it.

3. Choose a lower average in order to have few degrees below and more degrees above. They might choose 72°.

Record the temperatures again and compare each with 72°.

68° is 4° below 72°
74° - - - - - - - - - is 2° above 72°
77° - - - - - - - - - is 5° above 72°
4° below 7° above

Children should realize that the balance has changed. There are too many above and not enough below.

4. Children should discuss the estimates chosen, 72° and 74°. Note that the estimate was too large in one instance and too small in the other. Try 73°.

68° is 5° below 73°
74° - - - - - - - - - is 1° above 73°
77° - - - - - - - - - is 4° above 73°
5° below 5° above

Children note that 5 below balances 5 above.

Solution: 73° is the average temperature for the day.

5. Children should then find the average by adding the three numbers and dividing by 3.

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

1. Find the average of each of the following sets of numbers and check the solutions.

4, 5, 9, 10 60, 60, 70, 80, 90
6, 9, 12, 17 45, 60, 75, 85, 95

2. On our vacation trip we drove 432 miles the first day, 386 miles the second day and 364 miles the third day. How many miles did we average each day?

3. For a Scholarship Collection, 86 people contributed various amounts totaling $1978. What was the average amount contributed?

4. Discuss the meaning of averages in other situations: Test results, game scores, seasonal temperatures, rainfall statistics, etc.

5. Present other problems requiring children to find averages.

6. Look up the meanings of the word "average" in different dictionaries.

OPERATIONS

UNIT 68 - SET OF WHOLE NUMBERS: DIVISION; VERTICAL FORMAT

Objectives: To maintain skill in using the division algorithm.

To give practice in interpreting and then solving problem situations involving division.

TEACHING SUGGESTIONS

Maintain skill in dividing with divisors through 9.
Use oral drills, games, patterns, etc.

Maintain skill in dividing with divisors through 99; Digit in units column less than 5. For example, $24\overline{)268}$; $53\overline{)718}$; etc.

Divisors Through 99: Digit in Units Column Greater Than 5

Children who have developed skill in dividing with divisors through 99 with units digit less than or equal to 5, should use more difficult divisors through 99, where the digit in units column is greater than 5.

1. Reinforce multiplying by 10 and multiples of 10.

   Suggested problem: At the end of the year, Class 5-1 helped to store textbooks in the library. Each shelf holds 27 books. How many shelves had to be reserved for 891 books.

   Have children interpret the problem. (How many twenty-sevens in 891?)

   Children record the problem in equation form: $n \times 27 = 891$.

To estimate quotient children may first write:	Estimate:
$10 \times 27 = 270$	
$20 \times 27 = 540$	$891 > 20 \times 27$; $n > 20$
$30 \times 27 = 810$	
$40 \times 27 = 1080$ (too large)	$891 < 40 \times 27$; $n < 40$; n is between 20 and 40.

Have children use division algorithm with first partial quotient of 30 to complete the computation.

$$\begin{array}{r|l} 27\overline{)891} & \\ \underline{810} & 30 \\ 81 & \end{array}$$

Children should record first partial quotient and product from above.

They continue to find and record partial quotients and products.

$10 \times 27 = 270$ (too large)

$5 \times 27 = 135$ (too large)

$4 \times 27 = 108$ (too large)

$3 \times 27 = 81$

$$\begin{array}{r|l} 27\overline{)891} & \\ \underline{810} & 30 \\ 81 & \\ \underline{81} & \underline{3} \\ & 33 \end{array}$$

Solution: n = 33. The number of shelves needed is 33.

2. Children should check solutions.

Have children refer to the original problem and interpret their answer.

> If we discovered that 33 shelves are needed and each shelf holds 27 books, how many books are there altogether?
> $33 \times 27 = n$

Discuss why multiplying by 27 checks dividing by 27. (Inverse Operation).

Does the number 33 answer the original question?

Suggested Division Exercises With Remainders

Exercise: Divide 1852 by 28

```
     66            66
 28)1852         x 28
    1680  60      528
     172         1320
     168   6     1848
       4  66      + 4
                 1852
```

or (28 x 66) + 4 = (20 x 66) + (8 x 66) + 4
= 1320 + 528 + 4
= 1852

Discuss verifying solutions when remainders are involved.
Why do we add 4? (Refer to Unit 39)

Answer: Quotient 66, Remainder 4

1. Solve the following divisions. Verify the solutions by multiplication.

   48)1687   □ x 56 = 2408   79)1829   □ x 87 = 5978

2. Extend to divisions with more difficult divisors; with larger dividends.

   39)8085   17)3013   59)31147   48)14712   etc.

   Children check their solutions.

3. Interpret some of the above division exercises in problem situations.

Dividing Larger Numbers

Ability to estimate quotients helps children to divide larger numbers.

1. Suggested Problem: 43)92,851

   Necessary Background: Ability to multiply by 1000 and by multiples of 1000.

   1000 x 43 = □
   2000 x 43 = □
   3000 x 43 = □
   4000 x 43 = □

   1000 x 43 = □
   2000 x 43 = □
   4000 x 43 = □
   8000 x 43 = □

   2000 x 43 = □
   4000 x 43 = □
   6000 x 43 = □
   7000 x 43 = □

Estimate: 1000 x 43 = 43,000; $43{,}000 < 92{,}851$
2000 x 43 = 86,000; $86{,}000 < 92{,}851$
3000 x 43 = 129,000; $129{,}000 > 92{,}851$

Therefore the quotient is between 2000 and 3000 and 2000 is our first partial quotient.

2. Children then compute as above and check the solution.

They should discover that the underlying meaning and method of dividing numbers does not change with the size of the numbers.

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

1. Study the problem and computation below. Then answer the questions.

Mary divided 856 by 24, and checked her work in two ways. Her thinking looked like this:

```
     35           720        24
 24)856           120      x 35
    720  30        16       120
    136           856       720
    120   5                 840
     16  35                  16
                            856
```

Solution: Quotient 35
Remainder 16 or 35 R 16

Explain each step in the solution and in each check.

In the division, which number is the dividend? The quotient? The divisor?

Explain the meaning of the 30; the 720; the 5; the 16; the 35.

In the multiplication, which number is the same as the dividend, the quotient, the divisor?

What is each called in the multiplication?

Explain how the multiplication can verify the division.

Explain how addition will have helped to verify the division.

Which of the two checks Mary used, do you think is better? Why?

2. The Parents Association wants to order 1494 bottles of soft drinks for their meetings for the entire year. If they are packaged in cartons that hold 6 bottles each, how many cartons should the parents order? (n x 6 = 1494)

3. The thrush migrates from Louisiana to Alaska, covering the distance of 4000 miles in about 30 days. About how many miles does it fly in one day? (30 x n = 4000)

4. The district library has 9000 books. New book shelves are to be ordered, each of which will hold 36 books. How many shelves will be needed?

5. If one pair of socks cost $.59, how many pairs can mother buy for $4.00? What does the remainder tell?

6. John said, "I weigh 98 lb. The British unit of weight for 14 lb. is called a stone. How many stones do I weigh?"

7. Jane filled 16 birthday baskets with 12 pieces of candy each. Four of her guests did not come to the party. Jane put their share equally into each of the other baskets. How many pieces of candy did each guest receive in her basket? (2 step problem)

8. The 36 children of our class will send a basket to John who is ill. They will include books which cost $3.30, games for $1.74, a box of stationery for $1.98 and $.90 worth of fruit. How much should each child pay as his share?
($3.30 + $1.74 + $1.98 + $.90) ÷ 36 = n

9. Joe's graduation is 9 weeks away. He is saving money to pay for his expenses. He needs $12.50 for a class ring, $3.50 for the class party, $1.00 for the class gift and $2.50 for extra expenses. How much should he save each week to meet these costs?

10. Sally wants to do all the 336 problems in her arithmetic book during her vacation. She has finished one half of them. How many must she do each day to finish the problems in the 12 days left?

11. There were 214 children in the 8 classes of the fifth grade at the beginning of the school year. A new housing development opened in December and 58 more children were registered in the grade. What was the new average size of the classes on the grade?

12. Our class was moved to another room. The children helped to carry the books. If 426 books were moved, what was the average number of books each child carried? (missing information)

13. All the schools of the district were having a dance festival. The cost of using the auditorium for the 3 evenings was $385. The cost of the costumes for the 28 schools was $1267. The charge for 28 buses to transport the children was $700. How much money did each school have to contribute to cover the total expenses?

* 14. Astronaut John Glenn took 88 minutes to complete one orbit around the earth.

If he was aloft for 4 hours and 55 minutes, how many orbits did he make?

If the distance traveled in each orbit was about 27,016 miles, how many miles did he cover in one minute?

SETS; NUMBER; NUMERATION

UNIT 69 - SET OF FRACTIONAL NUMBERS: HUNDREDTHS; COMMON AND DECIMAL FORMS

Objectives: To introduce concept of hundredths, fractional and decimal forms.

To develop relationships: units, tenths, fifths, etc. to hundredths; cents to one dollar.

To develop concept of whole number plus fraction (mixed form) in decimal form.

TEACHING SUGGESTIONS

Extend understanding of relationships among fractions to include fractions through ninetieths.

1. Children should draw line and other diagrams to discuss.

   $\frac{1}{20}$ as $\frac{1}{2}$ of $\frac{1}{10}$; $\frac{1}{30}$ as $\frac{1}{3}$ of $\frac{1}{10}$; to $\frac{1}{90}$ as $\frac{1}{9}$ of $\frac{1}{10}$

2. Suggested exercises:

   a. $\frac{1}{20}$ = What part of $\frac{1}{10}$ $\qquad \frac{1}{60} = \frac{1}{\square}$ of $\frac{1}{20}$ $\qquad \frac{1}{20} = \square \times \frac{1}{40}$

   $\frac{1}{40}$ = What part of $\frac{1}{20}$ $\qquad \frac{1}{60} = \frac{1}{\square}$ of $\frac{1}{30}$ $\qquad \frac{1}{20} = \square \times \frac{1}{60}$

   etc. etc. etc.

   b. $\frac{1}{3}$ of $\frac{1}{10} = \square$ $\qquad \frac{1}{3}$ of $\frac{1}{20} = \square$ $\qquad \frac{1}{3}$ of $\frac{1}{30} = \square$ etc.

   $\frac{1}{5}$ of $\frac{1}{10} = \square$ $\qquad \frac{1}{6}$ of $\frac{1}{10} = \square$ $\qquad \frac{1}{7}$ of $\frac{1}{10} = \square$ etc.

c. What part of $\frac{1}{30}$ is $\frac{1}{60}$?

Can you think of a way to derive $\frac{1}{80}$? $\left[\frac{1}{8} \text{ of } \frac{1}{10}\right]$

d. Insert the correct numeral to make true statements.

$\frac{2}{20} = \frac{\square}{10}$, $\frac{4}{20} = \frac{\square}{10}$, $\frac{6}{20} = \frac{\square}{10}$, etc.

$\frac{10}{20} = \frac{\square}{10}$, $\frac{12}{20} = \frac{\square}{10}$, etc.

$\frac{40}{40} = \square$ $\frac{20}{40} = \square$ $\frac{10}{40} = \square$ $\frac{60}{60} = \square$ $\frac{20}{60} = \square$ etc.

e. Draw a number line. Count on the number line by steps of one-twentieth from 0 to 1.

$\left[\frac{1}{20}, \frac{1}{10}, \frac{3}{20}, \frac{2}{100}, \text{etc.}\right]$

Then do the above changing to tenths as you proceed.

$\left[\frac{1}{20}, \frac{1}{10}, \frac{3}{10}, \frac{2}{10}, \text{etc.}\right]$

Concept of Hundredths

Extend the understanding of fractions through hundredths.
Relate hundredths to tenths.

Using Squared Material.

Consider a"hundred square"as one unit.

Children should use the hundred square to discover relationships between:

The unit (the whole square) and tenths; the unit and hundredths; tenths and hundredths; etc.

Children should recognize that:

The unit is made up of ten rows with 10 small squares on each row.

The unit is made up of 100 small squares.

Each row is $\frac{1}{10}$ of the unit.

Each small square is $\frac{1}{100}$ of the unit. Why?

Each small square is $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{100}$ of the unit.

Outline other fractional parts of the unit; e.g. 20 squares (using 2 rows).

Children should record names for this fractional part in terms of tenths; in terms of hundredths; (2 tenths, $\frac{2}{10}$, .2, 20 hundredths, $\frac{20}{100}$)

Proceed similarly with 3 rows, 7 rows, 9 rows, 10 rows, (the entire unit)

Discuss the entire unit as 10 tenths; 100 hundredths: $\frac{10}{10} = 1$; $\frac{100}{100} = 1$

Discuss 28 squares, 37 squares, 3 squares, etc. as 28 of the one hundred, 37 of the one hundred, or as $\frac{28}{100}$, $\frac{37}{100}$, etc.

Hundredths in Decimal Form

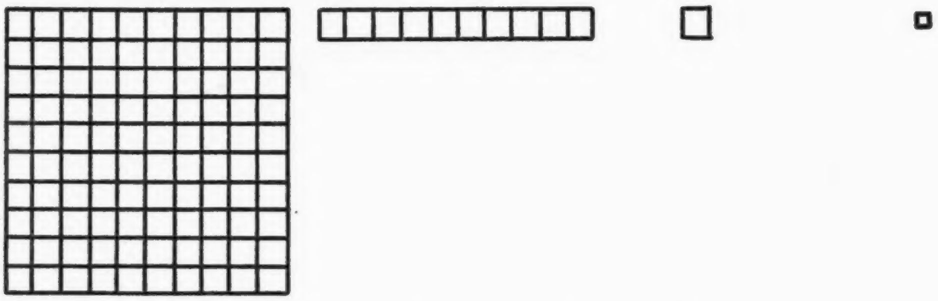
1. Place Value

Consider a small square as 1 unit.

a. Place a hundred square, a ten-square strip, a unit-square and a one-tenth strip (the result of dividing a unit-square into 10 equal parts) on the display board.

Draw a Place Value Chart directly below the squared material.

Record the numerals for the number of unit squares represented by the material on the Place Value Chart.



Hundreds	Tens	Units	Tenths
1	1	1	1

Cut one of the tenths into approximately 10 equal parts.

Ask children:

What should we call this part?
Where would you place this hundredth on the display board? [right of tenths place]
Why? [Each place has a value one tenth of the place immediately to its left.]
Where would you record the numeral for one hundredth on the chart?

H	T	U	Tenths	?
1	1	1	1	1

What would you call that place? [Hundredths] Why?

b. Children label the column "Hundredths".

They discuss the value of the 1 in the tenths place and the value of the 1 in the hundredths place.

H	T	U	Tenth	Hundredths
1	1	1	1	1

They note:

a "1" in the tenths place has 10 times the value of a "1" in the hundredths place.

a "1" in the hundredths place has $\frac{1}{10}$ the value of a "1" in the tenths place.

Children should discuss, then record 2 hundredths, 3 hundredths,... 9 hundredths on the place value chart.
Where should 10 hundredths be recorded?

Children show 3 hundredths on the place value chart.
They discuss how to record 3 hundredths without using a place value chart.

Record .03 on the chalkboard. Compare with .3.
What does the zero in .03 indicate?

Continue to 10 hundredths or .10 and compare with .1.

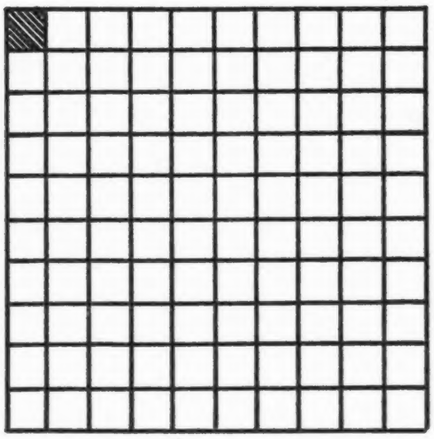
Continue to develop understanding and recording of more than 1 tenth or 10 hundredths.

Relationship: Units, Tenths, Fifths, etc. to Hundredths

Have children outline several "hundred-square" blocks on graph paper.
Have each hundred-square block represent one unit.

Children make comparisons by shading several fractional parts of the hundred-square blocks.

Relationship of One Unit to Hundredths



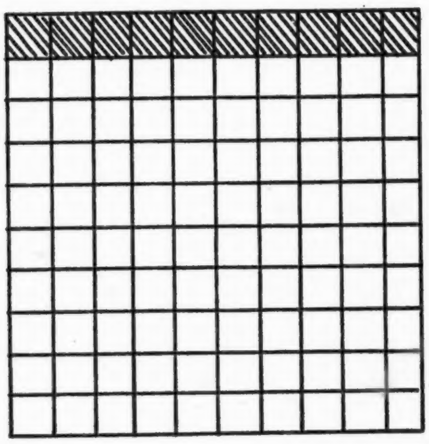
Have children shade one hundredth of the unit.

Compare the value of 1 unit with the shaded part.
Which is larger? How many times larger?
Each part is $\frac{1}{100}$ of the unit.
Each part is .01 or 0.01

Discuss the significance of the two zeros.

How many units are there?
How many tenths are there?

Relationship of Tenths to Hundredths



Have children shade one tenth of the unit.

Compare the value of 1 unit with the shaded part.
Which is larger? How many times larger?

$\frac{1}{10} = \frac{10}{100}$

$0.1 = 0.10$

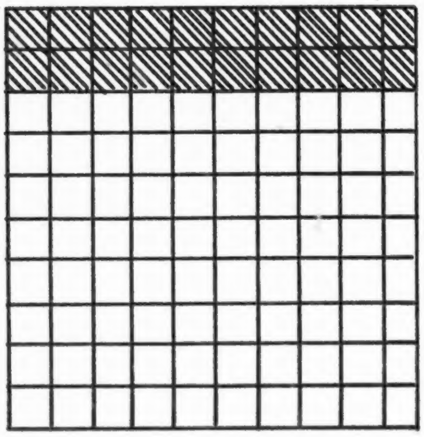
Have children shade additional hundred-square blocks to show:

$\frac{2}{10} = \frac{20}{100} = 0.2 = 0.20$

through

$\frac{10}{10} = \frac{100}{100} = 1 = 1.0 = 1.00$

Relationship of Fifths to Tenths; of Fifths to Hundredths



Have children shade on fifth of the unit.

Each part = $\frac{2}{10}$ of the unit.
Each part contains 2 tenths; $\frac{2}{10} = \frac{1}{5}$
Each part contains 20 hundredths

$\frac{1}{5} = \frac{2}{10} = \frac{20}{100}$

$\frac{1}{5} = 0.2 = 0.20$

Have children outline additional hundred-square blocks to show:

$\frac{2}{5} = \frac{4}{10} = \frac{40}{100}$ through

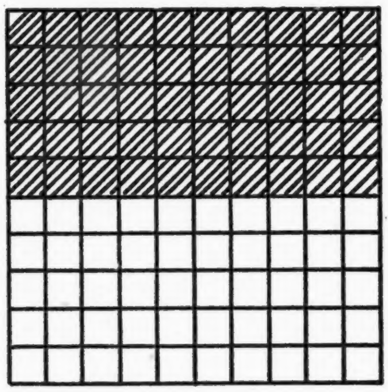
$\frac{5}{5} = \frac{10}{10} = \frac{100}{100} = 1$

$\frac{2}{5} = 0.4 = 0.40$ through

$\frac{5}{5} = 1 = 1.0 = 1.00$

Ask children: Do .6 and .60 name the same number? [Yes] Justify your answer. $\left[\frac{6}{10} = \frac{60}{100}\right]$

Relationship of Halves to Tenths; of Halves to Hundredths



Have children shade one half of the unit.

Discuss:

$\frac{1}{2} = \frac{5}{10} = \frac{50}{100}$

$\frac{1}{2} = 0.5 = 0.50$

$\frac{2}{2} = \frac{10}{10} = \frac{100}{100} = 1$

$\frac{2}{2} = 1 = 1.0 = 1.00$

Extend to other subdivisions; $\frac{1}{4}$, $\frac{1}{5}$, etc.

Relationship of Cents to One Dollar

1. Reinforce:

    1 cent as $\frac{1}{10}$ of 1 dime or 10¢
    1 dime (10¢) as $\frac{1}{10}$ of 1 dollar

    Use real money if necessary.

2. Suggested Exercises:

    1 cent = $\frac{1}{\square}$ of a dime.

    1 dime has a value of ___ times as large as 1 cent.

    1 dime = $\frac{1}{\square}$ of a dollar

    1 dollar = ___ dimes

    1 dollar has a value of ___ times as large as 1 dime.

3. Children know that one dollar = 100 cents and are now ready to understand and use such relationships as 1 cent is 0.01 of a dollar; a dime is 0.10 of a dollar; etc.

Children derive relationships such as:

1 cent = $\frac{1}{100}$ of a dollar = \$.01

1 nickel = $\frac{5}{100}$ of a dollar = \$.05

1 dime = $\frac{10}{100}$ of a dollar = \$.10

1 quarter = $\frac{25}{100}$ of a dollar = \$.25

1 half dollar = $\frac{50}{100}$ = \$.50

1 dollar = $\frac{100}{100}$ of a dollar = \$1.00

Have children write decimal symbols for the following amounts of money:

$\frac{3}{100}$ of a dollar; $\frac{75}{100}$ of a dollar; *$\frac{137}{100}$ of a dollar

Have children write the fraction symbols for:

\$.15 \$.27 \$5.00

Ask children:

Why do you think we use decimal symbols for money, as \$.20 rather than the fractional symbol $\frac{20}{100}$ of a dollar?

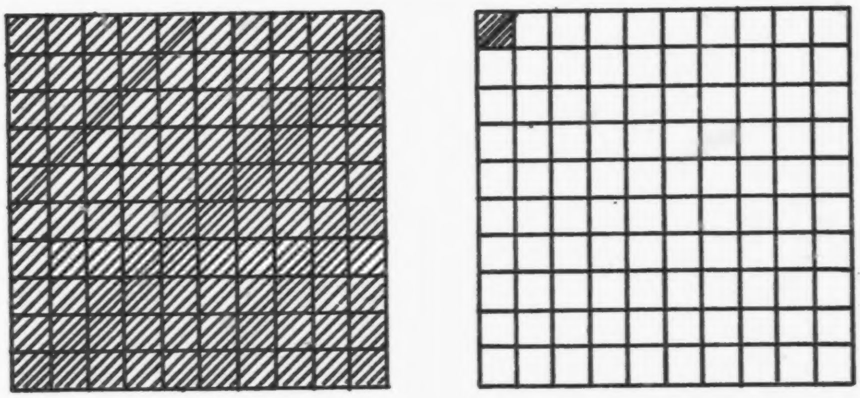
[It is easier to speak of, to write and to use in computations.]

*Some years ago, India changed to $\frac{1}{100}$ of a Rupee. (Optional)
England is planning to change, too.
Let children look this up in an Almanac, Encyclopedia, etc., and report on this information.
Have them explore Decimal System for money denominations in different countries.

Numbers In Mixed Form (Whole Number Plus Fraction): Common and Decimal Forms

Use 2 hundred-square blocks, one shaded to represent one whole, the other to represent hundredths of one whole.

Represent $1 \frac{1}{100}$ as illustrated.



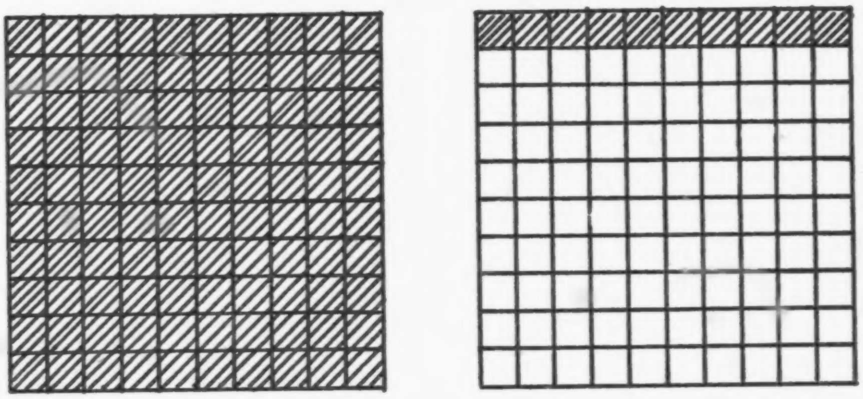
Children record the value represented as:

$1 + \frac{1}{100} = 1 \frac{1}{100}$

$1. + 0.01 = 1.01$

Continue through $1 \frac{9}{100}$

Represent $1 \frac{10}{100}$ as illustrated.



Children record the value represented as:

$1 + \frac{10}{100} = 1 \frac{10}{100} = 1.10$

$1 + \frac{1}{10} = 1 \frac{1}{10} = 1.1$

Explain: $1.10 = 1.1$

Follow a similar procedure with numbers such as:

$$1 + \frac{15}{100} = 1 + \frac{10}{100} + \frac{5}{100} = 1 + .10 + .05 = 1.15$$

$$1 + \frac{15}{100} = 1 + \frac{1}{10} + \frac{5}{100} = 1 + .1 + .05 = 1.15$$

EVALUATION AND / OR PRACTICE

SUGGESTED EXERCISES

1. Replace the frame or placeholder with a numeral to make true statements of each of the following:

.34 = 3 tenths + □ hundredths
.56 = □ tenths + 6 hundredths
*8 tenths + □ hundredths = $\frac{87}{100}$

.08 = △ tenths + 8 hundredths
4.97 = □ ones + △ tenths + ◇ hundredths
7 tenths + □ hundredths = $\frac{87}{100}$

2. Rename the following decimals in common fractional form.

4.05   8.17   28.27   60.06
6.75   9.60   13.30   27.25

3. Record the following in decimal form.

30 hundredths   14 and 3 tenths $\frac{9}{100}$ $23\frac{23}{100}$
7 and 36 hundredths   69 and 7 hundredths

4. Arrange the following in order of value from the smallest to the largest.

4.4   44   .44   4.44
.60   .24   .9   .53
.11   .90   .07   .1

5. Between each pair of decimal fractions insert > or < or = to form a true sentence.

.5   .45   5.5   .55
.7   .69   .62   .9
1.0   .99   .03   .3

OPERATIONS

UNIT 70 - SET OF FRACTIONAL NUMBERS: ADDING AND SUBTRACTING HUNDREDTHS; DECIMAL FORM

Objective: To help children develop the ability to add and subtract numbers involving hundredths, without and then with exchange; decimal form.

TEACHING SUGGESTIONS

Addition: No Exchange

Suggested Problem: The amount of rainfall in a month may be expressed in hundredths of an inch. Newton had 3.26 inches of rainfall in April and 1.71 inches in May. What was the total amount of rainfall for those two months? $3.26 + 1.71 = n$

Estimate: $3.26 > 3$;	$1.71 > 1$	$n > 4$	More than 4 inches of rainfall.
$3.26 < 4$;	$1.71 < 2$	$n < 6$	Less than 6 inches of rainfall.

Children should solve in a variety of ways, some of which are:

$3\frac{26}{100}$	$3.26 = 3 + .2 + .06$	3.26
$1\frac{71}{100}$	$1.71 = 1 + .7 + .01$	1.71
$4\frac{97}{100}$	$4 + .9 + .07 = 4.97$	4.97

Total amount of rainfall: 4.97 in.

Children should compare the sum with the estimated sum.

They should decide which algorithm is most efficient.

Addition: With Exchange

Suggested exercise: 2.56 + 3.78 = n

Children rewrite as:

$$\begin{array}{r} 2.56 \\ \underline{3.78} \end{array}$$

Method 1

Step 1 Have children:

Record the hundredths and add

$$\begin{array}{r} .06 \\ \underline{.08} \\ .14 \end{array}$$

Record the tenths and add

$$\begin{array}{r} .5 \\ \underline{.7} \\ 1.2 \end{array}$$

Record the whole numbers and add

$$\begin{array}{r} 2 \\ \underline{3} \\ 5 \end{array}$$

Step 2 Find the sum of hundredths, tenths and whole numbers.

$$\begin{array}{r} .14 \\ 1.2 \\ \underline{5} \\ 6.34 \end{array}$$

Solution: n = 6.34

Method 2

Children should examine the hundredths column.

They note:

.06 + .08 = .14

.14 = .10 + .04 They should record the 4 hundredths in the hundredths column.

$$\begin{array}{r} 2.56 \\ \underline{3.78} \\ 4 \end{array}$$

Since ten hundredths = one tenth, .10 is renamed as .1 to be added to the tenths.

They note the tenths column and add the tenths.

.1 + .5 + .7 = 13 tenths

13 tenths is renamed as 1.3. Why? They record the 3 tenths in the tenths column.

$$\begin{array}{r} 2.56 \\ \underline{3.78} \\ 34 \end{array}$$

The "1" is added to the whole numbers and the sum recorded.

$$\begin{array}{r} 2.56 \\ \underline{3.78} \\ 6.34 \end{array}$$

Children should observe that fractional numbers in decimal form are added just as whole numbers are added. They may verify the sum by changing to the common fractional form and/or using subtraction. Discuss the role and place of the decimal point.

Suggested Practice Exercises

1. Find the sum of each and verify each sum by changing to the common fractional form.

   (a) $\begin{array}{r} 39.25 \\ \underline{8.72} \end{array}$ (b) $\begin{array}{r} 115.37 \\ \underline{26.56} \end{array}$ (c) $\begin{array}{r} 135.92 \\ \underline{87.36} \end{array}$ (d) $\begin{array}{r} 94.86 \\ \underline{86.39} \end{array}$

   (e) $1.26 + 5.86 + 7.42$

2. Add 24.51, 43.09, 216.97

3. Find the sum for each set of numbers

   (a) 7.35, 12.68, 9.08
   (b) \$98.97, \$25.75, \$13.86
   (c) 29.08, 208.45, 8.67

4. Add: using the common fractional form. Convert to decimal form and verify your solutions.

   (a) $\begin{array}{r} 75\frac{1}{4} \\ \underline{26\frac{1}{2}} \end{array}$ (b) $\begin{array}{r} 93\frac{4}{5} \\ \underline{37\frac{1}{2}} \end{array}$ (c) $\begin{array}{r} 68\frac{3}{4} \\ \underline{72\frac{2}{5}} \end{array}$ (d) $\begin{array}{r} 56\frac{9}{10} \\ \underline{84\frac{7}{100}} \end{array}$

5. Carl added 32.14 and 26.2 as shown below and of course, his sum was incorrect.

$$\begin{array}{r} 32.14 \\ \underline{26.2} \\ 34.76 \end{array}$$

   Can you explain his error? (Hint: Use common fractions and expanded notation)

$$\begin{array}{l} 30 + 2 + \frac{1}{10} + \frac{4}{100} = 32.14 \\ 20 + 6 + \frac{2}{10} \qquad\quad = 26.2 \\ \hline 50 + 8 + \frac{3}{10} + \frac{4}{100} = 58.34 \end{array}$$

   What would you advise Carl to do when adding decimals?

Subtraction: No Exchange

Suggested Problem: Each foot in a surveyor's tape is divided into tenths and hundredths. Paul's father used a surveyor's tape to measure the length and width of the kitchen. The length was 10.63 feet and the width was 8.22 feet. How much greater than the width is the length of the kitchen? $10.63 - 8.22 = n$

Estimate: $10.63 > 10$; $8.22 > 8$; $.63 > .22$
The difference is greater than 2 feet.

Children should solve in a variety of ways, for example:

$$\begin{array}{r} 10\,\frac{63}{100} \\ -\ 8\,\frac{22}{100} \\ \hline 2\,\frac{41}{100} \end{array} \quad \text{or} \quad \begin{array}{rcl} 10.63 &=& 10 + .6 + .03 \\ -\ 8.22 &=& 8 + .2 + .02 \\ \hline && 2 + .4 + .01 = 2.40 + .01 = 2.41 \end{array} \quad \text{or} \quad \begin{array}{r} 10.63 \\ -\ 8.22 \\ \hline 2.41 \end{array}$$

Children should compare the exact difference with the estimated difference.

They should decide which algorithm is most efficient.

Subtraction: With Exchange

Suggested exercise: $9.62 - 3.45 = n$

Children should rename the minuend.

$$\begin{array}{rcccl} 9.62 &=& 9 + .6 + .02 &=& 9 + .5 + .12 \\ -\ \underline{3.45} &=& \underline{3 + .4 + .05} &=& \underline{3 + .4 + .05} \\ &&&& 6 + .1 + .07 = 6.17 \end{array}$$

Why did we change .6 to .5?

$$n = 6.17$$

Children should observe that fractions in decimal form are subtracted just as whole numbers are subtracted.

Suggested Practice Exercises

1. Verify remainders by using the common fractional form or by adding.

   (a) $\begin{array}{r} 9.69 \\ -\ \underline{3.27} \end{array}$ (b) $\begin{array}{r} 8.92 \\ -\ \underline{3.47} \end{array}$ (c) $\begin{array}{r} 18.04 \\ -\ \underline{3.92} \end{array}$ (d) $\begin{array}{r} 29.54 \\ -\ \underline{15.86} \end{array}$

   (e) From 8.34 subtract 2.93 (f) 127.00 minus 39.73

   (g) Subtract $\begin{array}{r} \$875.25 \\ \underline{350.98} \end{array}$ (h) Find the difference between 63.43 and 82.27.

2. Find the missing numeral.

   (a) $73.28 - n = 32.57$

   (b) $25.43 + n = 131.12$

   (c) $n + 524.85 = 913.72$

3. Subtract using the common fractional form. Convert to decimal form and verify your solution.

   (a) $\begin{array}{r} 15\frac{3}{4} \\ -\ 9\frac{1}{2} \\ \hline \end{array}$ (b) $\begin{array}{r} 28\frac{3}{5} \\ -\ 16\frac{1}{4} \\ \hline \end{array}$ (c) $\begin{array}{r} 85\frac{3}{10} \\ -\ 27\frac{7}{100} \\ \hline \end{array}$

4. Find the value of n.

   (a) $(27.3 + 6.9) - (8.5 + 3.6) = n$

   (b) $(42.8 + 5.7) - (16.2 + 4.8) = n$

   (c) $(127.5 + 32.8) - (29.3 + 15.7) = n$

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

ADDITION AND SUBTRACTION: COMMON AND DECIMAL FORMS

1. When Joan visited Barbara at the hospital, she walked $\frac{3}{4}$ of a mile to reach the hospital and $\frac{7}{10}$ of a mile from the hospital to her home. How many miles did she walk? ($\frac{3}{4} + \frac{7}{10} = n$)

2. It took Barbara $\frac{1}{4}$ hour to walk to the hospital from school. She stayed for $1\frac{1}{3}$ hours and then took $\frac{1}{2}$ hour to reach home. (Incomplete)

3. Linda has $1\frac{3}{8}$ yards of material. The pattern for the skirt she wants to make calls for $2\frac{1}{3}$ yards. How much more material must she buy.

4. A pail filled with a gallon of water weighs $10\frac{2}{15}$ lb. If the pail weighs $1\frac{8}{10}$ lb., how much does the gallon of water weigh?
($10\frac{2}{15} - 1\frac{8}{10} = n$)

5. The table in the clubhouse is $11\frac{1}{3}$ ft. long. We have one cloth $4\frac{1}{2}$ ft. long, another $7\frac{3}{4}$ ft. long. After the table is covered how much is left for overlap and drop at the ends?

6. A tank contained 92.4 gallons of oil. During the day, 15.75 gallons were used. How much oil remained in the tank?

7. With $85.00 in cash, Mr. Bates set out to pay two bills. One bill amounted to $25.15. The other was $19.75. After Mr. Bates paid these bills, how much cash did he have left?

8. In 1961, the winning racing car traveled at an average speed of 139.44 miles per hour. In 1962, the winning car traveled at an average speed of 142.29 miles per hour. What was the difference in speed?

OPERATIONS

UNIT 71 - SET OF FRACTIONAL NUMBERS: MULTIPLICATION; COMMON FORM; PROPERTIES APPLIED

Objective: To help children use properties of multiplication (Distributive and Commutative) in multiplication of fractions.

TEACHING SUGGESTIONS

Distributive Property Applied: Suggested Development

Problem: John wants to make a book shelf. He needs $2\frac{1}{3}$ feet of wood for each shelf. How much lumber will he need for 4 shelves? ($4 \times 2\frac{1}{3} = n$)

Estimate: $n > 8$; $n > 9$ (Why?) [$4 \times \frac{1}{3} > \square$]

Compute: Children may solve in a variety of ways. They should observe that the Distributive Property is applied.

Horizontal Format

A. $4 \times 2\frac{1}{3} = (4 \times 2) + (4 \times \frac{1}{3}) = 8 + \frac{4}{3} = 8\frac{4}{3} = 9\frac{1}{3}$

B. $4 \times 2\frac{1}{3} = (4 \times \frac{1}{3}) + (4 \times 2) = \frac{4}{3} + 8 = 9\frac{1}{3}$

C. $$\begin{array}{l} 4 \times 2 = 8 \\ 4 \times \frac{1}{3} = \underline{\ \frac{4}{3}\ } \\ \qquad\quad 8\frac{4}{3} = 9\frac{1}{3} \end{array}$$

D. $$\begin{array}{l} 4 \times \frac{1}{3} = \frac{4}{3} \\ 4 \times 2 = \underline{8} \\ \qquad\quad 8\frac{4}{3} = 9\frac{1}{3} \end{array}$$

Vertical Format

$$\text{E.}\quad \begin{array}{rl} 2\frac{1}{3} & \\ \underline{\times\, 4} & \\ 8 & (4 \times 2) \\ \underline{\frac{4}{3}} & (4 \times \frac{1}{3}) \\ 8\frac{4}{3} = 9\frac{1}{3} & \end{array} \qquad\qquad \text{F.}\quad \begin{array}{rl} 2\frac{1}{3} & \\ \underline{\times\, 4} & \\ \frac{4}{3} & (4 \times \frac{1}{3}) \\ \underline{8} & (4 \times 2) \\ 8\frac{4}{3} = 9\frac{1}{3} & \end{array}$$

Children may check by adding. $2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} = 8\frac{4}{3} = 9\frac{1}{3}$

Suggested Practice:

$3 \times 4\frac{1}{5} = 12 + \frac{\square}{5} = n$ $\qquad$ $5 \times 3\frac{2}{3} = 15 + \frac{\square}{3} = n$

$6 \times 4\frac{1}{9} = \square + \Delta = n$ $\qquad$ $4 \times 5\frac{2}{7} = \square + \Delta = n$

$5 \times 2\frac{5}{6} = 10 + \frac{\square}{6} = n$

$3 \times 6\frac{5}{12} = \square + \Delta = n$

Commutative Property of Multiplication Applied: Suggested Development

1. Reinforce the generalization for multiplication of fractions. (Multiplying the numerator of the fraction by the whole number gives the numerator of the product. The denominator remains the same.)

   a. Have children solve:

   $4 \times \frac{1}{5} = \square$ $\qquad$ $\frac{1}{5} \times 4 = \square$

   Ask children:

   Since $4 \times \frac{1}{5} = \frac{4}{5}$, and $\frac{1}{5} \times 4 = \frac{4}{5}$, then $4 \times \frac{1}{5} = \frac{1}{5} \times \square$

b. Have children solve:

$\frac{3}{4} \times 8 = \square$ $\qquad$ $8 \times \frac{3}{4} = \square$

Ask children:

Since $\frac{3}{4} \times 8 = 6$, and $8 \times \frac{3}{4} = 6$, then $\frac{3}{4} \times 8 = 8 \times \square$

c. Ask children to explain the property involved.

2. Have children complete the following to make each statement true. Explain.

$8 \times \frac{1}{6} = \frac{1}{6} \times \square = n$ $\qquad$ $\frac{1}{8} \times 46 = 46 \times \square = n$

$\square \times \frac{2}{3} = \frac{2}{3} \times 5 = n$ $\qquad$ $\square \times 60 = 60 \times \frac{7}{10} = n$

$12 \times \frac{3}{10} = \square \times 12 = n$ $\qquad$ $\frac{2}{9} \times 84 = \square \times \frac{2}{9} = n$

$35 \times \square = \frac{7}{12} \times 35 = n$ $\qquad$ $\frac{3}{20} \times \square = 95 \times \frac{3}{20} = n$

Since $2 \times 4\frac{2}{3} = 9\frac{1}{3}$, $\quad 4\frac{2}{3} \times 2 = \square$ $\quad$ Why?

Since $6 \times 2\frac{3}{10} = 13\frac{4}{5}$, $\quad 2\frac{3}{10} \times 6 = \square$ $\quad$ Why?

Multiplication: Changing Mixed Form to Fractional Form

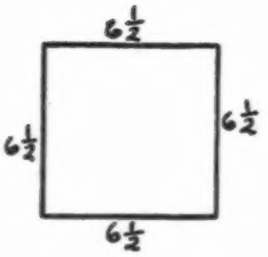
1. Suggested Development

Problem: Each side of John's square garden plot is $6\frac{1}{2}$ ft. long. How much fencing must he buy to build a fence around it.

Reinforce the formula for the perimeter of a square. $\left[\begin{array}{l} P = S + S + S + S \\ P = 4S \end{array}\right]$

Have children write the sentence to solve the problem. $\left[4 \times 6\frac{1}{2} = n\right]$

Reinforce: $6\frac{1}{2} = \frac{\square}{2}$



Rewrite the problem: $4 \times 6\frac{1}{2} = n$ as $4 \times \frac{13}{2} = n$

Then compute: $4 \times \frac{13}{2} = \frac{52}{2} = 26$

2. **Present problem:** Mother needs to fill $2\frac{3}{4}$ books of trading stamps. If each book contains 1200 stamps, how many stamps does she need to collect?

Method I: $2\frac{3}{4} \times 1200 = (\square \times 1200) + (\Delta \times 1200) = n$ (Applying Distributive Property of Multiplication with respect to Addition)

Method II: $2\frac{3}{4} \times 1200 = \frac{11}{4} \times 1200 = n$ (Changing Mixed Form to Fractional Form)

EVALUATION AND/OR PRACTICE
SUGGESTED EXERCISES

1. Children compute the following using methods 1 and 2.

   a. $35\frac{2}{3} \times 900 = n$
   b. $56\frac{3}{8} \times 1432 = n$

2. Complete the following:

   a. $5\frac{2}{3} \times 6 = \frac{\square}{3} \times 6 = n$
   b. $4 \times 2\frac{1}{8} = 4 \times \frac{\square}{8} = n$
   c. $8\frac{3}{4} \times 10 = \frac{\square}{4} \times 10 = n$
   d. $4\frac{1}{5} \times 18 = \frac{\square}{5} \times 18 = n$
   e. $7 \times 4\frac{2}{3} = 7 \times \frac{\square}{3} = n$
   f. $6 \times 1\frac{5}{16} = 6 \times \frac{\square}{16} = n$

3. Complete and find the product.

$$\begin{array}{r} 8\frac{2}{3} \\ \underline{\times\ 6\quad\quad} \\ (6 \times \square) \\ \underline{(6 \times \Delta)} \end{array}$$

4. Find the product $5 \times 7\frac{3}{8}$ by renaming $7\frac{3}{8}$ as a sum.

5. Interesting Number Patterns

Show that the following statements are true:

$$1\frac{1}{2} \times 3 = 1\frac{1}{2} + 3$$

$$1\frac{1}{3} \times 4 = 1\frac{1}{3} + 4$$

$$1\frac{1}{4} \times 5 = 1\frac{1}{4} + 5$$

Can you continue the pattern?

OPERATIONS

UNIT 72 - SET OF FRACTIONAL NUMBERS: FRACTION AS AN INDICATED DIVISION OF TWO NUMBERS; DIVISION OF FRACTIONS

NOTE TO TEACHER

Fraction As A Number

The sets of numbers that we consider in Elementary School Mathematics are:

1. The Set of Counting Numbers (also called Natural Numbers)
   $\{1, 2, 3, \ldots\}$
2. The Set of Whole Numbers
   $\{0, 1, 2, \ldots\}$
3. The Set of Integers
   $\{\ldots, {}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, \ldots\}$
4. The Set of Rational Numbers

Rational numbers may be defined as numbers which can be symbolized as an indicated quotient, $\frac{a}{b}$, of two integers (the denominator not zero). Thus one-half, which can be written as $\frac{1}{2}$ is a rational number, as in three, which can be written as $\frac{3}{1}$ (or $\frac{6}{2}$).

Fraction As A Numeral

Technically, a fraction, like $\frac{1}{2}$ or $\frac{2}{1}$ is a numeral, naming or symbolizing a rational number.
It is understood that we will use the term "fraction" whether we are considering the number (the idea) or the numeral (the symbol).
Children need rarely refer to this distinction in grade 5.

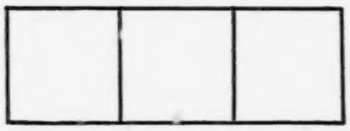
Objectives: To extend the meaning of fractional numbers.
To develop dividing a fraction by a whole number.

TEACHING SUGGESTIONS

Fraction As An Indicated Quotient

1. Suggested problem: 3 boys are to share a candy bar equally. What part of it will each boy get?

Show, using a diagram, how the boys shared the candy bar.



What did the boys do to the candy bar? [They divided it into 3 equal parts.]

What part did each boy get? $\left[\frac{1}{3}\right]$

Ask children to record the action in a mathematical sentence.
$\left[1 \div 3 = \frac{1}{3}\right]$

Is $\frac{1}{3}$ another symbol for $1 \div 3$? Can $\frac{1}{3}$ be read as $1 \div 3$?
What can the line separating the numerator and the denominator represent?

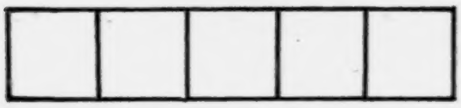
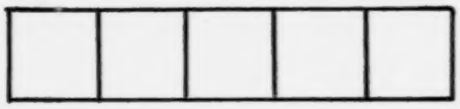
[the bar of the division symbol;

• ←numerator
—←indicated division
• ←denominator]

2. **Suggested problem:** 2 candy bars were to be divided equally among 5 boys. How much should each boy get?

Ask children:

To draw a diagram to show the action.

To write a mathematical sentence to show the action.

$\left[2 \div 5 = \frac{2}{5}\right]$

What is the result of the division represented by $2 \div 5$ or $\frac{2}{5}$

$\left[\frac{2}{5} \text{ or 2 fifths}\right]$

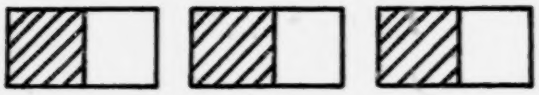
To read $\frac{2}{5}$ in two different ways.

Children discover that:

$\frac{2}{5}$ read as $2 \div 5$ represents the result of the operation of division on 2 and 5.

$\frac{2}{5}$ read as 2 fifths represents the result of that operation on the answer.

3. **Suggested problem:** 3 candy bars are to be divided equally between 2 boys. Show this with a diagram and record the action with symbols.



$\left[3 \div 2 = \frac{3}{2}\right]$

Have children interpret $\frac{3}{2}$ in two different ways.

4. Using number lines and symbols.

Have children draw a line segment of 1 unit length.

They divide it into 2 equal parts and label the midpoint.

Write the sentence that tells how you arrived at 1 half the unit of length. $1 \div 2 = \frac{1}{2}$

Is $\frac{1}{2}$ the same as $1 \div 2$? Why?

Is $1 \div 2$ another name for $\frac{1}{2}$? Why?

Have children draw a line segment of 3 units of length.

They divide it into 2 equal parts.

They show that $3 \div 2 = \frac{3}{2}$ and that $\frac{3}{2}$ is another name for $3 \div 2$.

5. Suggested exercises for practice.

   a. Complete the following equations and change to mixed form where possible:

   $1 \div 6 = \frac{\square}{\triangle}$ $\frac{\square}{\triangle} = 4 \div 5$ $7 \div 3 = \frac{\square}{\triangle}$ $25 \div 8 = \frac{\square}{\triangle}$

   $\frac{3}{5} = \square \div \triangle$ $\frac{5}{10} = \square \div \triangle$ $100 = \square \div \triangle$ $\frac{400}{100} = \square \div \triangle$

   b. What operation does the symbol for a fraction indicate?

   c. State another interpretation of a fraction. [A fraction indicates the quotient of two numbers.]

Dividing a Fraction by a Whole Number

1. Suggested problem: $\frac{1}{4}$ of a large candy bar is divided equally between 2 children. How much will each receive?

Children may use representative materials if necessary.

Discuss:

What part of the candy bar must be shared? $\left[\frac{1}{4}\right]$

Into how many parts must $\frac{1}{4}$ of the candy bar be divided? [2]

To have children understand how to record the division of a fraction by a whole number use the following pattern:

Ask children to record:

8 divided into 2 equal parts; $2\overline{)8}$; $8 \div 2$

4 divided into 2 equal parts; $2\overline{)4}$; $4 \div 2$

2 divided into 2 equal parts; $2\overline{)2}$; $2 \div 2$

1 divided into 2 equal parts; $2\overline{)1}$; $1 \div 2$

$\frac{1}{2}$ divided into 2 equal parts; $2\overline{)\frac{1}{2}}$; $\frac{1}{2} \div 2$

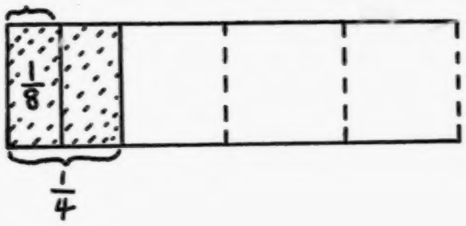
$\frac{1}{4}$ divided into 2 equal parts, $2\overline{)\frac{1}{4}}$, $\frac{1}{4} \div 2$

How then may the problem be recorded in a mathematical sentence? $\left[\frac{1}{4} \div 2 = n\right]$

What will "n" equal? $\left[n = \frac{1}{8}\right]$

Solution: $\frac{1}{4} \div 2 = \frac{1}{8}$

Have children derive or check answers by drawing diagrams.



2. Suggested problem: For the school garden, $\frac{1}{3}$ of a plot of land is to be shared equally by 4 classes. What part of the entire plot will each class cultivate? $\frac{1}{3} \div 4 = n$

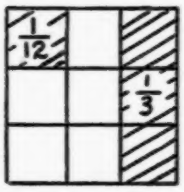
Discuss:

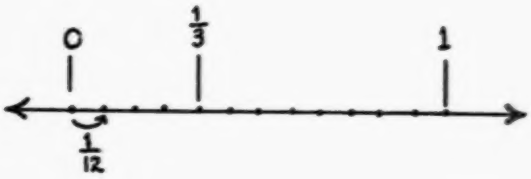
What part of the plot must be shared?

Into how many parts must $\frac{1}{3}$ of the plot be divided?

Record: $\frac{1}{3}$ divided into 4 parts = n

Children should represent the situation using a diagram or a number line.





$\frac{1}{3}$ divided into 4 equal parts = $\frac{1}{12}$ n = $\frac{1}{12}$

3. Suggested practice exercises.

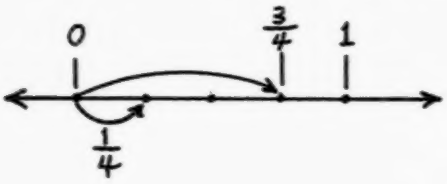
Children may use diagrams or number rays to derive the answer.

$\frac{1}{2} \div 2 = n$ $\frac{1}{4} \div 2 = n$ $\frac{1}{2} \div 3 = n$ $\frac{1}{4} \div 3 = n$, etc.

Dividing a Fraction With a Numerator Greater Than One by a Whole Number.

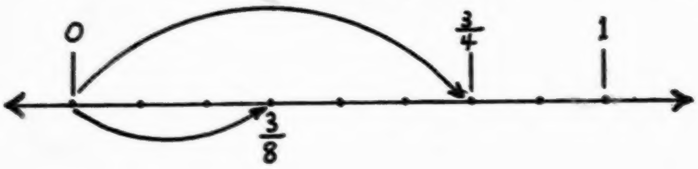
1. Suggested Exercise: $\frac{3}{4} \div 3 = n$

Discuss: $\frac{3}{4}$ divided into 3 equal parts. Use diagrams or number line.



$\frac{3}{4} \div 3 = \frac{1}{4}$

2. **Suggested problem:** $\frac{3}{4}$ of a yard of ribbon is to be divided into 2 equal parts for apron strings. How long will each string be? $\frac{3}{4} \div 2 = n$



Discuss:

$\frac{1}{4}$ divided into 2 equal parts equals $\square$? $\left[\frac{1}{8}\right]$

$\frac{3}{4}$ divided into 2 equal parts equals $\square$? $\left[\frac{3}{8}\right]$ Why?

$\left[\frac{3}{4}\right.$ is 3 times as much as $\left.\frac{1}{4}\right]$

3. **Suggested practice exercises:**

Since $\frac{1}{3} \div 3 = \frac{1}{9}$ Since $\frac{1}{6} \div 2 = \frac{1}{12}$ Since $\frac{1}{5} \div 2 = \frac{1}{10}$

$\frac{2}{3} \div 3 = \square$ $\frac{5}{6} \div 2 = \square$ $\frac{4}{5} \div 2 = \square$

SETS; NUMBER; NUMERATION

*UNIT 73 - SYSTEMS OF NUMERATION: BASE FIVE (Optional)

NOTE TO TEACHER

The Hindu-Arabic system of numeration is a decimal or base 10 system.
It uses 10 symbols called digits:

{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

and 10 is called the Base.
Any whole number can be recorded using a combination of these ten symbols and the principle of place value. This can be extended to include fractional numbers expressed as decimal fractions.

A similar system of numeration can be based on any number of symbols greater than 1.
For example:

Base	No. of Symbols	Symbols
2	2	0,1 (Binary System of Numeration)
3	3	0,1,2
4	4	0,1,2,3
5	5	0,1,2,3,4 (Quinary System of Numeration)
10	10	0,1,2,3,4,5,6,7,8,9 (Decimal System of Numeration)
11	11	0,1,2,3,4,5,6,7,8,9,T
12	12	0,1,2,3,4,5,6,7,8,9,T,E (Duodecimal System)

The reason for studying a system of numeration based on another number such as 2 or 5 is to give children a better understanding of the Base 10 system and the use of the Place Value concept.

When introducing and developing the brief study of the Base 5 system of numeration suggested below, the emphasis should be only on understanding and appreciation and on comparison with the decimal (Base 10) system. Emphasis should not be on computation, speed, skill or memorization.

Objective: To extend understanding of the decimal system of numeration by introducing numeration in Base 5.

TEACHING SUGGESTIONS

1. Reinforce characteristics of the decimal system of numeration. Discuss:

   The names of the symbols (digits)

   The number of symbols

   Name of the system [Decimal system of numeration] Why?

   How numbers greater than 9 are recorded

   Place Value - Use the Place Value Chart to record and explain the value of the digits in each column. For example:

H	T	O
4	3	6

   Tell children that the decimal system of numeration is also called a Base 10 system. Why?

2. Introduce Base Five

   Develop with children:

   What would you call a system similar to our Base 10 system, with only five symbols? [Base 5 or Quinary System]

   Have children make up their own symbols. For example:

   Δ, □, *, O, X

   If we use Hindu-Arabic symbols, which of them should we use? [0, 1, 2, 3, 4] Why?

   How would you represent the numbers one, two, three, four? [1, 2, 3, 4]

   What would the next number be? [five]

   In the Base 10 system, have we a single symbol for ten, the base?

   How do we represent ten? [10]

In the Base Five system, we would represent five by 10_{five}. There is no single symbol for five in Base Five, just as there is no single symbol for ten in Base 10.

We read 10_{five} as "one, zero; Base 5"

Discuss:

The verbalization of "10" when it represents the quantity five. [One, zero; Base Five]

The reason why "one, zero; Base Five" should not be called ten. [It is too easily confused with the number 10.]

The recording of "one, zero; Base Five" to distinguish it from "one, zero; Base 10."

10_{five}	read as:	One, zero; Base Five
10	read as:	Ten. (Base need not be indicated in Base Ten, although it may also be read as "One, zero; Base Ten)

Continue to discuss, record and read Base Five quantities through 9.

Number	Numeral	Read as
five	10_{five}	one, zero; base five
six	11_{five}	one, one; base five
seven	12_{five}	one, two; base five
eight	13_{five}	one, three; base five
nine	14_{five}	one, four; base five

How shall we continue?

3. Place Value Chart for Base Five

Discuss how to label the columns. Begin with "ones." Have children volunteer. Teacher records symbols for column labeled "Ones." They record numbers through 4.

	Ones
	1
	2
	3
	4

Extend chart to the next column.

> In Base Ten system what did we label the column to the left of unit's column? Why?
> [Ten times as large as the one's unit]
>
> In Base Five system what can we label the column to the left of "One's" column? [Fives]
> Why? [Five times as great as the "One's" unit]

Have children:

> Record "Five" in the Place Value Chart.
>
> Tell what is the number after 4. [five]
>
> Record it in its proper place.

Fives	Ones	
	1	
	2	
	3	
	4	
1	0 (one, zero)	(one group of five, 𝍸)

Discuss:

> Number of ones, number of groups of fives. Explain.
> [Four is the largest number that can be represented in the ones column]

What is the relationship between 10_{five} and 1_{five}?

[10_{five}: read as one, zero; is five times the size of 1_{five}.]

4. Comparing numerals in Base Five with numerals in Base 10

a. Use chart to make comparisons:

BASE FIVE		BASE TEN or DECIMAL EQUIVALENT	
Fives	Ones		
	1	1	𝍷
	2	2	𝍷𝍷
	3	3	𝍷𝍷𝍷
	4	4	𝍷𝍷𝍷𝍷
1	0	5	𝍸

b. Have children continue to record numerals in Base Five and to compare them with the decimal equivalents.

$11_{five} = 6_{ten}$; $12_{five} = 7_{ten}$; $13_{five} = 8_{ten}$; $14_{five} = 9_{ten}$

BASE FIVE		DECIMAL EQUIVALENT			
Fives	Ones				
1	0	5	𝍸		
1	1	6	𝍸	𝍷	
1	2	7	𝍸	𝍷𝍷	
1	3	8	𝍸	𝍷𝍷𝍷	
1	4	9	𝍸	𝍷𝍷𝍷𝍷	
2	0	10	𝍸	𝍸	(2 sets of five)

Discuss:

20_{five} [Two, zero; Base Five represents 2 sets of five and no sets of one, which is equal to 10 in Base 10.]

Continue through 44 (24) [Represents 4 sets of five and 4 sets of one]

c. Extend the Place Value Chart.

Relate the Place for 10 tens (hundreds) to the Place for 5 fives.

Record 25 in Base Five (100). Children should see the need for another column.

BASE FIVE			DECIMAL EQUIVALENT
Twenty-fives	Fives	Ones	
	4	4	24
1	0	0	25

What is the largest quantity that can be shown in two columns in the decimal system? [99]
Why? There is no single symbol to represent a value larger than 9.

What is the largest quantity that can be shown in two columns in the quinary system? [44]
Why? [We cannot use the symbol 5 in Base Five]

d. Discuss relationship between column headings in the Quinary System.

Fives and ones; ones and fives
[Five times as large]
[One fifth as large]

Twenty-fives and fives; fives and twenty-fives

Twenty-fives and ones; ones and twenty-fives

e. Compare these relationships with relationships in the decimal system; tens and ones; hundreds and tens; etc.

Discuss:

Value of 42_{five} as 4 fives and 2 ones. [(4 x 5) + 2, in base 10]

Method of changing 42_{five} to Base Ten.

$$42_{\text{five}} = (4 \times 5) + 2 = 22 \text{ in Base Ten}$$

Have children change the following numerals in Base Five to numerals in Base Ten.

33_{five} 24_{five} 101_{five}

5. Suggested practice exercises:

a. Write each of the following Base Five numerals as a Base Ten numeral.

BASE FIVE	BASE TEN
0	[0]
10	[5]
24	[14]
42	[22]
103	[28]

b. Construct a chart to compare numbers from zero to fifty in Base Five with numbers from zero to fifty in Base Ten.

c. Insert the correct relation symbol: >, =, <

$17 \square 11_{\text{five}}$ [>] $29 \square 120_{\text{five}}$ [<]

$19 \square 34_{\text{five}}$ [=] $1000 \square 131_{\text{five}}$ [>]

*d. Some children may be able to add and subtract in Base Five.

*e. Encourage children to develop numeration systems with other bases.

GEOMETRY AND MEASUREMENT

UNIT 74 - MEASUREMENT: GRAPHIC REPRESENTATION; LINE GRAPH

Objectives: To interpret Line Graphs.
To construct Line Graphs.

TEACHING SUGGESTIONS

Reading and Interpreting Line Graphs

1. Reinforce:

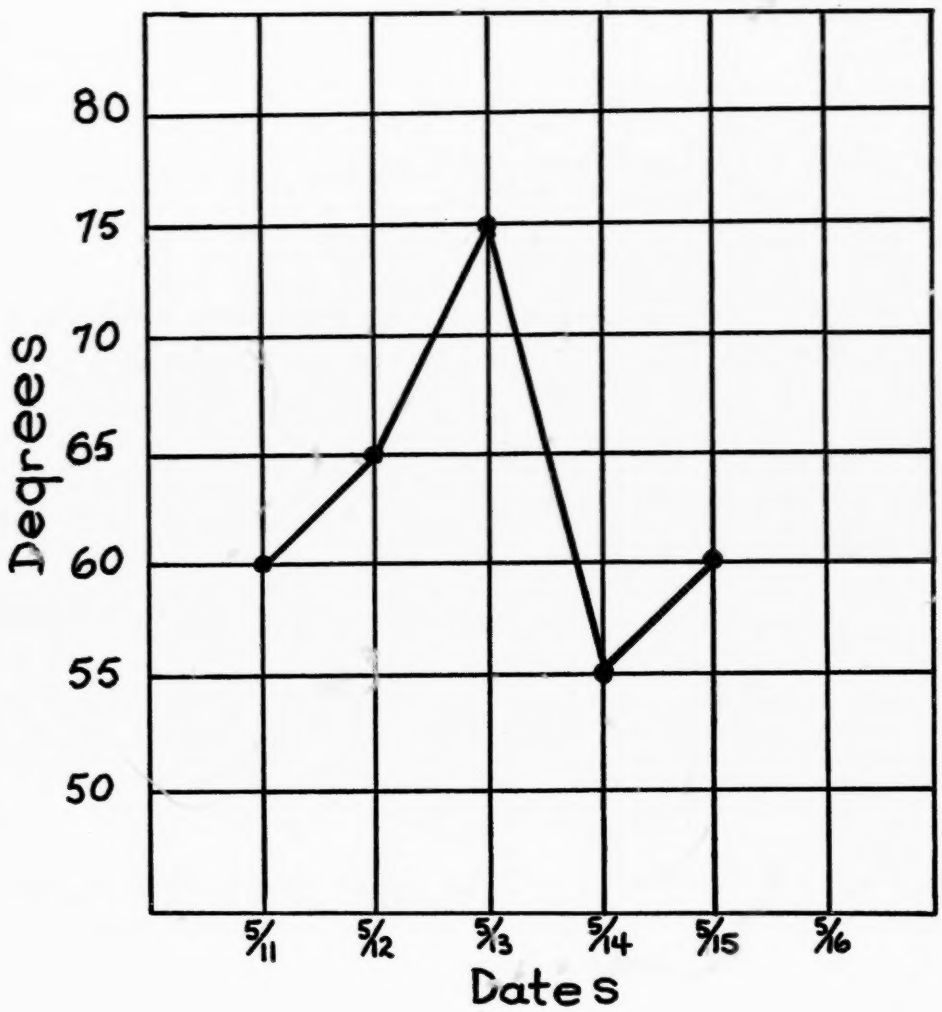
   Interpretation of bar graphs as showing a way of comparing data.

   Meaning of vertical axis, horizontal axis, scale.

2. Present a graph as below, and discuss the following:

   What is the title of the graph?
   What do the dates at the bottom tell us?
   What do the numerals at the left tell us?
   On what day or date was it warmest? - coldest?
   On what dates was the temperature the same?
   How much warmer was it on May 13th than on May 11th?
   How many degrees did the temperature drop between May 13th and May 14th?

Record of 2 P.M. Temperature—Week of May 11th



Discuss the advantage of using this Line Graph rather than a Bar Graph.

Line graphs are useful to show general trends and changes as well as to facilitate comparisons. A line graph may show changes over a period of time; changes in weight as heights change; etc.

3. Study line graphs and note:
   - a. A line graph should have a title.
   - b. Each axis should be labeled.
   - c. The graph has two axes.
   - d. A line graph scale does not necessarily start at zero.
   - e. The lines are numbered, not the spaces.
   - f. The mark used to show data on a line graph is a "dot" which will become the endpoint of one or two line segments.
   - g. The dot represents an ordered pair. On the chart above each day is paired with the average temperature at 2 P.M. on that day.

     On the graph above each dot represents one of the ordered pairs: 12, 65; 15, 60; etc.
   - h. Dots are connected by line segments from left to right. The line segments make it easier to interpret the graph and to observe a trend.
   - i. When one of the scales of a line graph represents units of time, these are usually marked on the horizontal axis.

4. Children should interpret other line graphs found in newspapers; periodicals and textbooks.

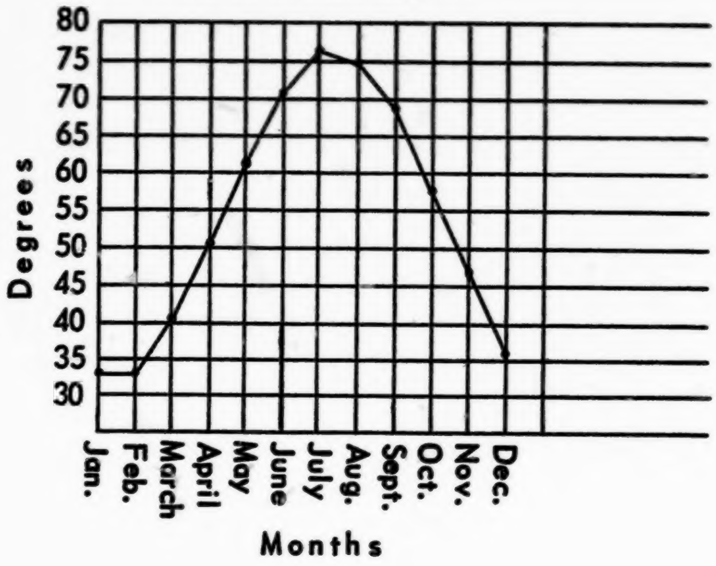
Constructing Line Graphs

1. Children should use graph paper (4 or 5 boxes to the inch are suggested) to construct a line graph from information organized on a chart.

For example:

Normal Monthly Temperatures for New York City

Month	Temp.	Month	Temp.
Jan.	33	July	77
Feb.	33	Aug.	75
Mar.	41	Sept.	69
April	51	Oct.	58
May	62	Nov.	47
June	71	Dec.	36


Normal Monthly Temperatures for New York City

As children construct the graph of the information on the chart, discuss:

> Why was a scale of 5° for each space selected for the degrees of temperature?
>
> What other scale could have been chosen? Explain.
>
> Why were the months written on the horizontal axis?

What determined the placement of the dots?
[Ordered pairs as shown on chart]

During what 2 months was there no change in temperature?

If there is a sharp rise upward in the line from 1 month to the next, what does that tell about the increase?

Compare the use of a bar graph vs. a line graph for this data.

2. Have children construct and interpret other line graphs based on information from daily experiences, textbooks, etc.

GEOMETRY AND MEASUREMENT

UNIT 75 - MEASUREMENT: AREA OF A RECTANGULAR REGION

NOTE TO TEACHER

In measuring area by counting the number of units, we follow the same pattern as when we measured length by counting hand spans, etc.

We:

1. Select a unit.
2. Count the number of units that fit into the object being measured.
3. This number is called the measure; in this case, the area.

Objectives: To develop concept of area.
To introduce standard units of measure for area.
To develop formula for finding area.

TEACHING SUGGESTIONS

1. Reinforce meaning of point, line, line segment, ray, plane, simple closed figure, polygon.

2. To understand area as the measure of the interior surface of a polygon, children should review properties of some geometric figures.

a. Reinforce: Right Angle
   Angle: Meaning of angle; kinds of angles.
   Right Angles are formed when 2 intersecting lines result in 4 angles with equal measure.
   Perpendicular lines are 2 lines that intersect to form right angles.

b. Reinforce properties of Rectangles.
   A rectangle is a polygon with 4 sides (quadrilateral).
   Both pairs of opposite sides are parallel.
   Both pairs of opposite sides are equal in length.
   All four angles are right angles.
   A rectangle encloses a region called a Rectangular Region.

2. Develop measurement of a Rectangular Region.

a. Discuss the need for measuring a rectangular region: for example, the amount of paper, tile, carpet, etc. needed to cover a surface.

Ask children:

What is meant by the "play area" of the playground?
What is meant by the "floor space" of a room?
How do you think you can find the measure of an enclosed region?

Tell children:
The measure of an enclosed space (the region) is called its area. Area measures surface just as length measures curves or lines.
It can sometimes be found by counting the number of times a selected unit is contained in the region being measured.

b. Non-Standard Units of Measure of Area.

Suggested problem: To find the area of the drawing paper.

Provide each child with various shapes: circular, triangular, square, etc.

Consider each in turn as a unit of area.

(The square should be contained an exact number of times in the sheet of drawing paper.)

Have children experiment with the various shapes to find how many units of any one of them are contained within the surface.

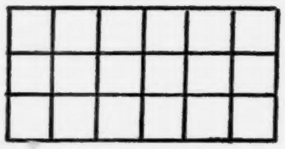
Discuss the fact that to measure the surface area, the units must not overlap, and spaces must not be left between units.

Which unit of area was most convenient for measuring the surface? [the square unit] Why?

Which unit of area was impossible to use for measuring the surface? [the circle] Why?

Have children lay off the square unit and count the number of times the unit is repeated to cover the whole surface.

How many times was the square unit contained in 1 row across the top of the drawing paper?



For how many rows was this repeated?

If 6 square units were laid off horizontally for 3 rows, how many square units were contained on the surface?

Have children discover that the area is 6 square units repeated 3 times, or 3 times 6 square units, or 18 square units.

Children discover that the area is 6 x 3 unit squares.

Suggested problem: To measure a section of the floor using a square unit.

Have children determine the number of times the length of the square is contained in 1 horizontal row, and the number of rows needed.

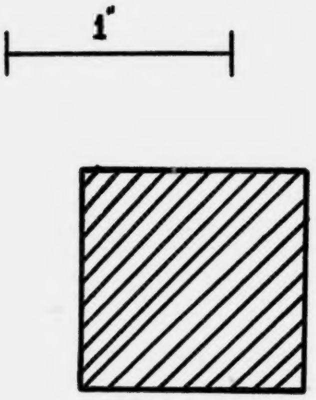
Discuss various ways of finding the area of the section of the floor by:

Counting the number of units the section of the floor will contain.
Multiplying the number of units in each row by the number of rows.

c. Standard Units of Measure

Square Inch

Discuss:



One inch as a unit of length.

A one inch square as enclosing 1 square inch of area.

The enclosed region has an area of 1 square inch.

One square inch as a standard unit of measure for area.

How to construct a square inch.

Difference between a "1 inch square" and "1 square inch".

[In a "1 inch square" we are talking about the square, which is the boundary; in "1 square inch" we are talking about the measure of the interior region of a "1 inch square".]

Suggested problem: Find the area of a sheet of drawing paper measuring 9" x 12".

Have children:

Mark the paper into one inch squares.

Count to find the number of square inches in one row and the number of rows.

Find the area by counting the number of one inch squares that the paper contains. [108 inch squares]

Discover that they can also find the area in square inches if they multiply the number of inches in one row (length) by the number of rows (width) 9 x 12 = 108. [108 square inches]

Square Foot

Discuss:

one foot as a unit of length.
one square foot as a unit of area.

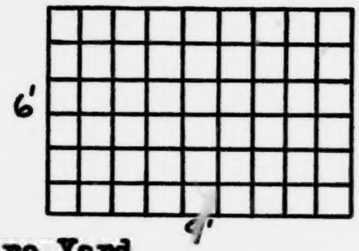
How would you construct a square foot using a foot rule? Using the square inch?

How many square inches are there in a square foot? [144] Why?

What is the area in a 1 foot square in square inches? in square feet?

Suggested problem: What is the area in square feet of a dinette floor that measures 6' by 9'?

Have children draw a diagram to show the area marked off in square feet.



Find the area:
6 x 9 sq. ft. = 54 sq. ft.

What does the 6 represent? the 9?

Square Yard

Discuss need for using larger units of square measure, such as square yard, to measure larger surfaces, e.g. carpeting a floor.

Children discover that:

144 sq. in. = 1 sq. ft. Explain. - Note that $144 = 12 \times 12 = 12^2$

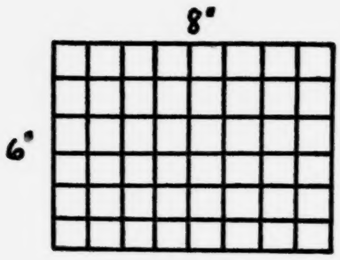
9 sq. ft. = 1 sq. yd. Explain. - Note that $9 = 3 \times 3 = 3^2$

1296 sq. in. = 1 sq. yd. Explain. - Note that $1296 = 36 \times 36 = 36^2$

Discuss why 1296 = 9 x 144.

3. Formula for finding Area of a Rectangular Region

Consider a rectangle 8" long and 6" wide.



Have children:

Count the number of one inch squares in one row. [8]

Measure the length of the row. [8 inches]

Relate the measure of the length (8 inches) to the number of inch squares. [8; the number is the same.]

Count the number of rows. [6; width]

Measure the width. [6 inches]

Relate the measure of the width (6 inches) to the number of rows. [6; the number is the same]

Arrive at the formula for finding the area of a rectangular region:

If A = Area; l = length; w = width

then $A = l \times w$ or $A = lw$

Apply the formula to find the area of the rectangular region above.

$A = l \times w$ $A = 8 \times 6$ $A = 48$ Area = 48 sq. inches

4. Discuss:

Need for square mile as a unit to measure large areas of land.

Acre as a unit of area. (Not "square acre" since acre is already a unit of area.)

Relationship of area of square to square numbers.

Need for using same unit of measure of length in finding areas.

Children should understand that when the dimensions are stated in two different units of measure, such as feet and inches, either the number of larger units may be changed to smaller units, or the number of smaller units may be renamed as fractional parts of the larger unit.

Children should note that when:

There are more of the smaller units; the number of those units is larger.

There are fewer of the larger units; the number of those units is smaller.
For example: 1 ft. = 12 in., $1 < 12$

EVALUATION AND / OR PRACTICE

SUGGESTED EXERCISES

1. Find the area of the following rectangular regions.

   12' by 15'   14" by 14"   25 yd. by 32 yd.

   5' by 8'9"   8' by 6 yd.   4'3" by 2'7"

2. Additional exercises may be found in textbooks.

GEOMETRY AND MEASUREMENT

UNIT 76 - GEOMETRY: SOLID GEOMETRIC SHAPES

Objectives: To help children distinguish between plane and solid figures.

To distinguish solids by their characteristics

TEACHING SUGGESTIONS

1. Reinforce children's understanding of the following terms.

Line
Line segment
Ray
Parallel

Perpendicular
Surface
Region
Dimensions

Angle
Right angle
Obtuse angle
Acute angle

Vertex
Simple closed figure
Plane
Polygon

Parellelogram
Quadrilaterial
Square
Rectangle

Circle
Arc
Degree

Area

Trapezoid
Rhombus
Perimeter

2. Discuss the meaning of dimension.

Children should know intuitively that:

A point has no dimensions.

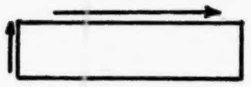
A line segment has one dimension; length

A closed plane figure such as a rectangular region has two dimensions; length and width.

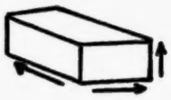
Compare a plane surface, e.g. a desk top with a solid, e.g. individual cereal box.

Have children note:

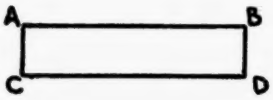
The boundaries of the desk top are line segments.
The boundaries of the box are parts of planes.



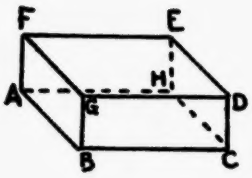
That the desk top has length and width.
That the box has length, width, and depth; the box encloses a volume.



The number of corners (vertices) of the desk top. (4)



The number of corners (vertices) of the box. (8)



That both are closed figures.

Discuss the dimensions of the desk top and the dimensions of the box.

Tell children that the box is an example or model of a geometric solid.

Show children:

filled cereal box; empty cereal box
brick; empty chalk box

Are all of these models of geometric solids? Why?

Discuss:

3 - D films
Photographs as plane figures (2 dimensional) that suggest solid or 3 dimensional figures.

3. Characteristics of some solid geometric figures.

Display items that are examples of solids:

Orange, ball, globe, pyramid, cylinder or can, building block, cube, triangular prisms, cone, etc.

Identify each as an example of a solid and compare them.

Note that some solids:

Have spherical shapes. (orange and ball have curved surface only.)

Have curved and plane surfaces. (cylinder, cone)

Have only plane surfaces. (block, pyramid, prism)

Continue the development using only the following:

Brick, block, a square box, pyramid, triangular prism.

Emphasize the similarities of each. (Each has surfaces that are enclosed by polygons).

Discuss:

The surfaces as faces

The line segments as edges

The intersections of the line segments as vertices

Emphasize the differences

Compare a cube with other rectangular solids. (boxes)

Compare a pyramid with the other solids.

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Show diagrams of plane and solid figures. Identify each.

2. Find and list objects in the classroom that suggest geometric solids.

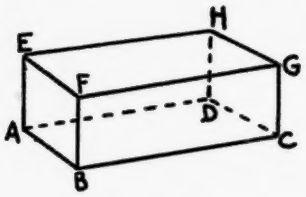
3. What geometric figure is suggested by each of the following:

    can
    carton
    ice cream cone
    ball
    pup tent
    tepee

4. Write the name of an object which suggests each of the geometric terms listed below:

    sphere
    rectangular solid
    cube
    cone

5. Draw a picture of a rectangular solid. Label each vertex with a letter.



Identify each edge by two letters. [AB, etc.]

Identify each face by four letters. [EFGH, ABFE, etc.]

Identify each vertex by one letter. [A, B, etc.]

Use letters to identify the faces of the solid that have the largest area, the smallest area.

Use letters to identify the edges that are parallel. [$\overline{AB} \parallel \overline{DC}$, etc.

6. Indicate which of the following suggest surface regions only; which represent geometric solids:

    an ice cube

    a can of peas

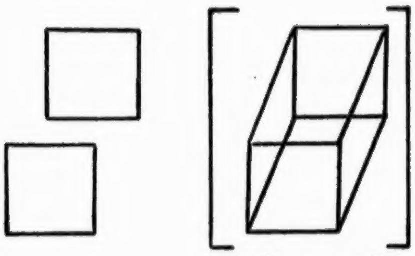
    a counter top

    the outside of the can of peas

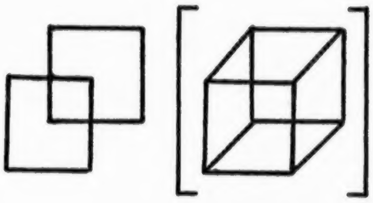
Visualizing, Drawing, Constructing of 3 dimensional figures may help children to understand better their properties.

1. Starting with the following, can you finish the drawing to make a model of

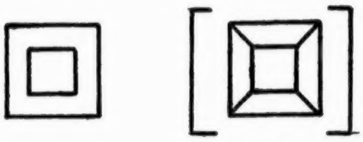
    a. A cube:



Not a cube

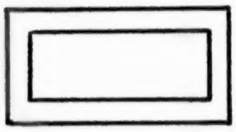


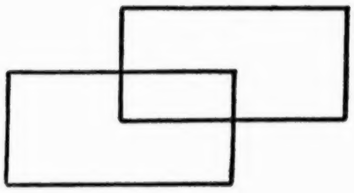
A cube

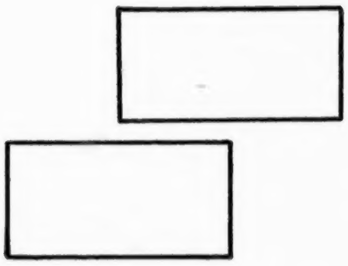


A cube

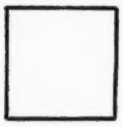
    b. A rectangular solid





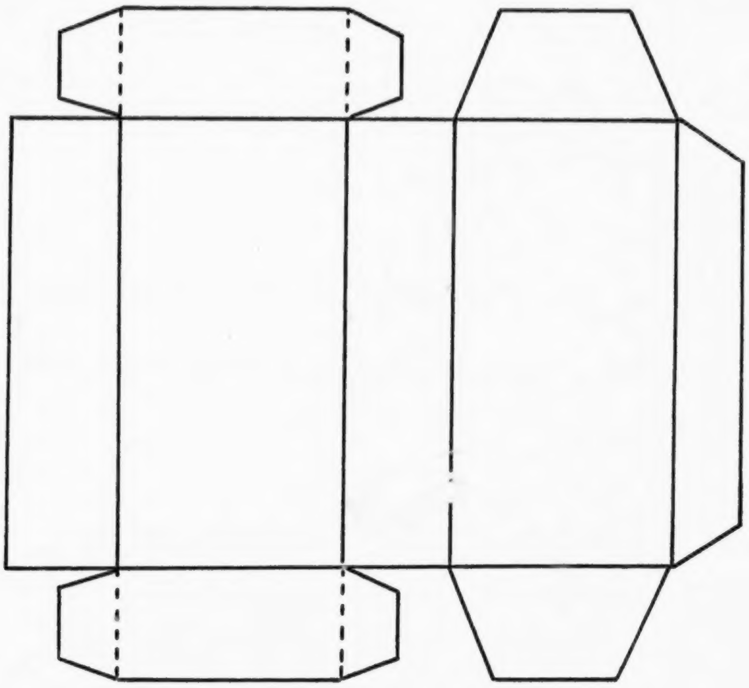


c. A pyramid: (starting with a square and a point outside the square)



2. Visualize and then draw a picture of some 3 dimensional figures as they would look flattened out.

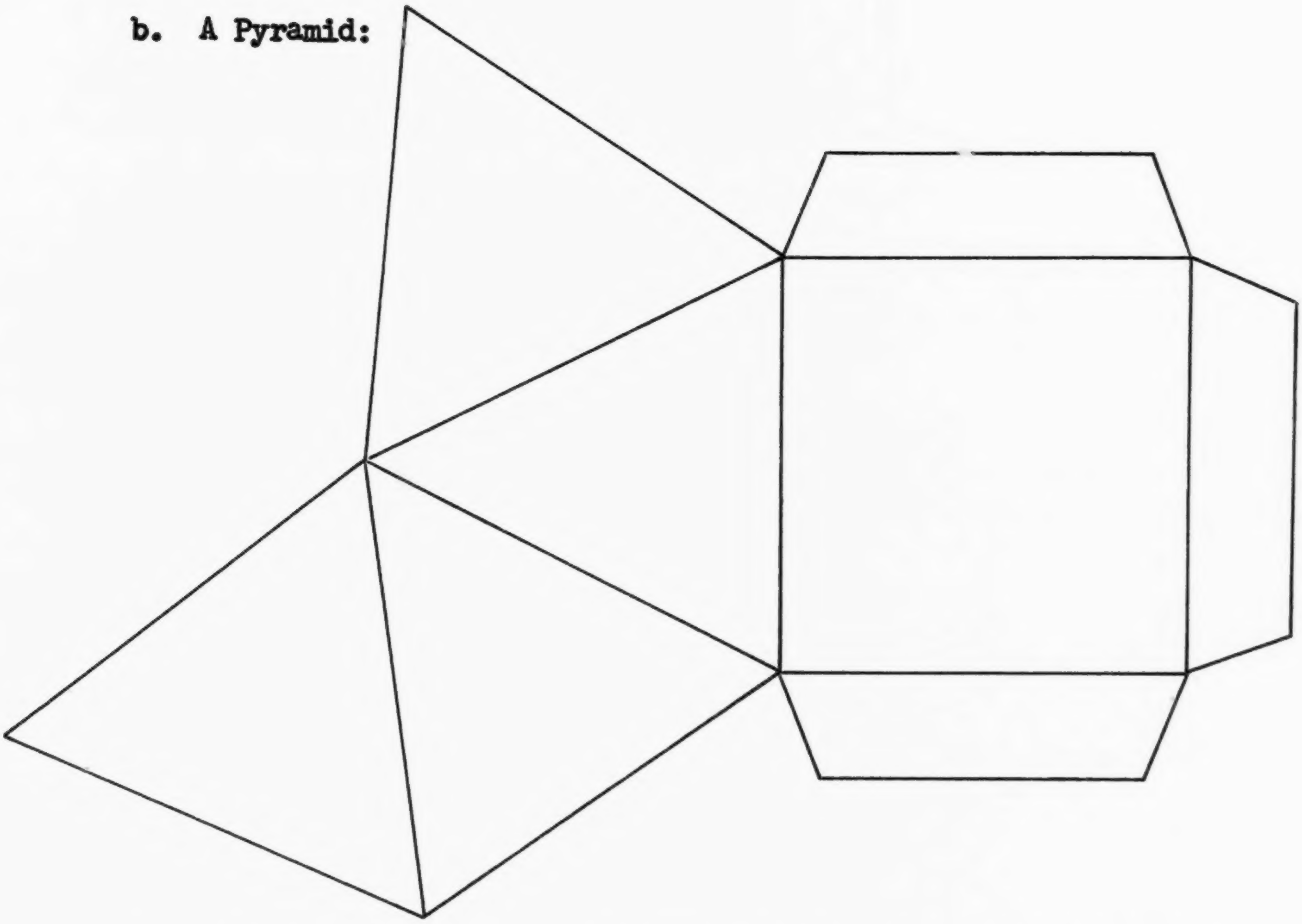
   a. A Rectangular solid:



Describe a rectangular solid as you visualize it from the flattened out model.

Add tabs to your model, cut it out and construct the model of a rectangular solid.

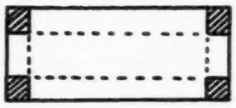
b. A Pyramid:



Describe a pyramid as you visualize it from the flattened out model.

Add tabs to your model, cut it out and construct the model of a Pyramid.

c. Start with a rectangular sheet of paper or card board.



Cut squares of the same dimensions out of each corner.

Make an open box by folding up the four sides. Use tape to connect the sides.

Experiment with cutting squares of different sizes out of each corner. Describe your results.

Experiment with cutting rectangles out of each corner. Describe your results.

SETS; NUMBER; NUMERATION

UNIT 77 - NUMERATION: EXPONENTIAL NOTATION

NOTE TO TEACHER

Children should be aware that in scientific notation, numbers are expressed using exponents and powers of 10. For example:

$10{,}000 = 10 \times 10 \times 10 \times 10 = 10^4$ can be read as: Ten to the fourth power.

10 is referred to as the "base". The term "base" has different meanings in systems of numeration, (Base 10, Base 5) and in Geometry.

For 10^4, the 4 is referred to as the Exponent. The exponent 4 tells how many times the base 10 is used as a factor.

10^4 is called the fourth power of 10.

Zero, used as an exponent is a special case.

$10^0 = 1$, by definition as is a^0 for any $a \neq 0$

One, used as an exponent is a special case.

$10' = 10$, by definition: $a' = a$ for all "a".

Objective: To help children understand meaning and use of exponents.

TEACHING SUGGESTIONS

Exponent for Squares

1. **Reinforce understanding of the meaning of the square of a number.**

Ask children:

What are the two equal factors that result in the product

25 [5 x 5]   36 [6 x 6]   64 [8 x 8]

1 [1 x 1]

What number multiplied by itself equals 100?
How many times does 5 appear as a factor of 25? [2 times]
6 as a factor of 36? [2 times] etc.

2. **Tell children that:**

When the same factor is used 2 times to arrive at a product, the expression may be written as:

$5 \times 5 = 25$ or 5 square = 25 or $5^2 = 25$

$6 \times 6 = 36$ or 6 square = 36 or $6^2 = 36$

The ^{2} in 5^2, 6^2, etc. is called the exponent;

The 5, 6 is the factor that is being repeated.

This type of recording may be called the exponential form.

5^2 is called the second power of 5.

25 can be written as:

25 in Whole Number Form

5 x 5 in Product Form

5^2 as a power of 5 or in Exponential Form

3. **Suggested practice exercises:**

a. Express the following as the product of 2 equal factors, and then in exponential form.

1, 4, 16, 49, 81, 100

b. Complete the following, then state in exponential form.

9 = □ x □ = □ square = ____ [3^2]
121 = □ x □ = □ square = ____ [11^2]
400 = □ x □ = □ square = ____ [20^2]

c. 12^2 = □　　13^2 = □　　14^2 = □　　15^2 = □

d. If $n^2 = 100$ then n = □
If $n^2 = 121$ then n = □
If $n^2 = 144$ then n = □
If $n^2 = 169$ then n = □

* e. If one side of a square is 4 inches, what is the area of the square? Write this in exponential form.

* f. If we let one side of a square be "s", write a formula for the area of a square, using exponential form. [$A = S^2$]

Extended Exponential Notation

1. Discuss:
In how many ways can we name 16 as a product?
Record some of these ways. For example:

a. 16 x 1　　b. 8 x 2　　c. 4 x 4　　d. 4 x 2 x 2
e. 2 x 2 x 2 x 2

In which of the above are only equal factors used?

What are the factors of 16 in c? [4 x 4]; in e? [2 x 2 x 2 x 2]

How many times is the factor 4 used in c? the factor 2 used in e?

Rename the following, repeating the same factor:

25 = __ x __ [5 x 5]
64 = __ x __ [8 x 8]
64 = __ x __ x __ [4x4x4]
27 = __ x __ x __ [3x3x3]
81 = __ x __ x __ x __ [3x3x3x3]
100 = __ x __ [10x10]

Ask children to consider 25 as 5 x 5.
Which factor is repeated? [5]
How many times is it used as a factor? [2 times]
How can we write this in exponential form? [5^2]

Ask children to consider 81 as 3 x 3 x 3 x 3
Which factor is repeated? [3]
How many times is it used as a factor? [4 times]

2. Tell children that 3 x 3 x 3 x 3 can be written as 3^4.

3^4 can be read "three to the fourth power".

Ask children:

What do you think the 3 in 3^4 means? The factor that is to be repeated.

What do you think the ^{4} in 3^4 means? [The number of times 3 is used as a factor.]

If this form of renaming numbers is called exponential notation, which number do you think we should call the repeated factor? [3]; the exponent? [4]; the base? [3]

Read 6^2 in two different ways. [6 to the second power, or 6 square]
Read 5^3 in two different ways. [5 to the third power, or 5 cube]
Why do we call it "6 - square"; "5 - cube" ?

3. Provide practice in reading numerals written in exponential notation.

3^5 , 8^3 , 7^4 , etc.

4. Provide practice in renaming numbers by repeating a factor, then stating the exponential form.

e.g. $4 = 2 \times 2 = 2^2$, $8 = 2 \times 2 \times 2 = 2^3$, etc.

5. Express in words and tell the meaning of:

23^1 , 43^1 , n^1 [In n^1 we cannot think of "n used as a factor once", so what we do is to define n to equal n.]

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Fill in the frames with the correct numeral.

$8 = 2 \times 2 \times 2 = 2^{\square}$ [2^3] $25 = 5 \times 5 = \square^2$ [5^2]

$49 = 7 \times 7 = 7^{\square}$ [7^2] $512 = 8 \times 8 \times 8 = \square^3$ [8^3]

$10{,}000 = 10 \times 10 \times 10 \times 10 = 10^{\square}$ [10^4]

$32 = 2 \times 2 \times 2 \times 2 \times 2 = \square^{\triangle}$ [2^5]

2. Express each of the following in words and tell the meaning of:

8^2 [eight to the second power, or eight square]

5^3 [five to the third power, or five cube]

6 [six to the seventh power]

3. Write the following in exponential form and tell the value of each:

9 to the fourth power. [$9^4 = 9 \times 9 \times 9 \times 9 = 6561$]

Repeated factor 4, exponent 2 [$4^2 = 16$]

Repeated factor 10, exponent 3 [$10^3 = 1{,}000$]

3 to the fifth power [$3^5 = 243$]

4. Solve

$3^4 = \_\_ \times \_\_ \times \_\_ \times \_\_ = n$ [$n = 81$]

$4^3 = \_\_ \times \_\_ \times \_\_ = n$ [$n = 64$]

$10^6 = \_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_ = n$ [$n = 1{,}000{,}000$]

5. **Solve**

$64 = 4^{\square}$ [4^3] $81 = 9^{\square}$ [9^2]

$64 = 2^{\square}$ [2^6] $81 = 3^{\square}$ [3^4]

6. **Fill in the spaces in the following chart:**

Word Name	Numeral	Repeated Factor Form	Exponential Form
Twenty-five	25	5×5	5^2
One hundred twenty-five		$5 \times 5 \times 5$	
	625	$5 \times 5 \times 5 \times 5$	
Sixty-four	64		
Eight			

7. **Which of the following represents the largest number? the smallest number? Explain.**

53, 3^5, 5^3,   35, 5×3, $5 + 3$

[largest: $3^5 = 243$; smallest: $5 + 3 = 8$]

8. **Which number is larger? How much larger?**

3^4 or 4^3

5^2 or 2^5

2^4 or 4^2

9. Extend the Place Value Table - Base Five to include exponential notation.

One Hundred Twenty-fives	Twenty-fives	Fives	Ones
125	25	5	1
5 x □ x □	5 x □	5	1
5	5	5	1

SETS; NUMBER; NUMERATION

*UNIT 78 - NUMBER: SET OF INTEGERS (Optional)

NOTE TO TEACHER

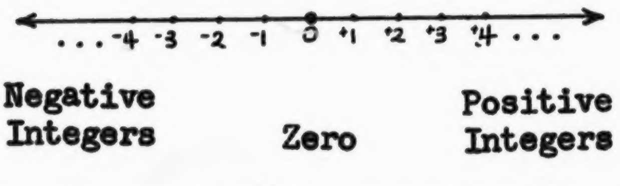
Children have worked with the set of Counting or Natural Numbers $\{1, 2, 3, \ldots\}$ and with the set of Whole Numbers $\{0, 1, 2, \ldots\}$. We are ready now to extend the number system to include the set of Integers.

The Set of Integers consists of:

> The Set of Counting Numbers, Zero, and for each number, n, of the set of Counting numbers, another number, ^-n, such that:

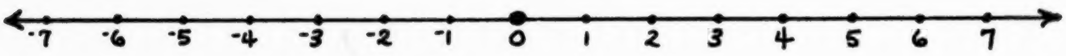
$$n + {}^-n = 0$$

The Set of Integers



A number line represents a one-to-one correspondence between a set of numbers and their associated points on a line. It may be a horizontal line, a vertical line or a line in any other direction extending indefinitely in both directions.

Positive and negative integers may be represented by evenly spaced dots on the number line to the right and to the left (or above and below) the point that designates the number Zero.



6 shown as $^{+}6$ is read "positive 6"

8 shown as $^{+}8$ is read "positive 8"

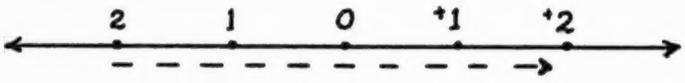
When the sign of the number is not shown, it is understood that the number is positive. $8 = {}^{+}8$; $6 = {}^{+}6$; etc.

$^{-}4$ is read "negative 4"

$^{-}3$ is read "negative 3"

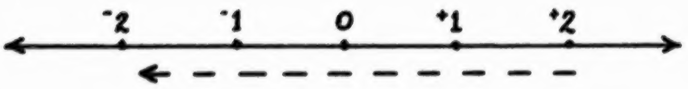
Zero is a number which is neither positive nor negative. It is the reference point or origin on the number line.

On the number line, positive numbers usually are to the right of the zero. The positive direction here, indicates that we proceed to the right from any point on the number line.



Positive Direction

The Negative direction indicates that we proceed to the left from any point on the number line.



Negative Direction

We sometimes call integers "directed numbers" because of the way we represent them on the number line. Often the term "signed Numbers" is used because of the $^+$ or $^-$ sign in the numeral.

Objectives: To introduce the meaning of and symbolism for integers.

To introduce concept of negative and positive direction on the number line.

TEACHING SUGGESTIONS

Set of Integers: Concepts

1. Reinforce reading a thermometer. Consider the thermometer as a vertical number line.

   Have children discuss and record temperatures above zero, below zero, at zero.

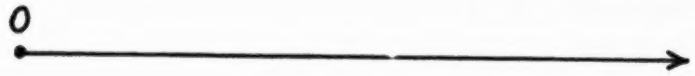
   Ask children:

   What is the origin or reference point on the number line? [zero]

   How many times does zero appear on the number line? [one]

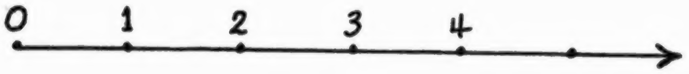
   How many times does every other numeral appear on the number line? [2 times] Where?

2. **Have children draw a horizontal Number Ray and indicate zero.**



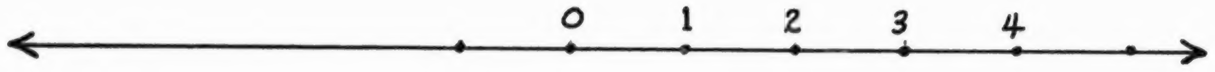
They should then mark off unit to the right of zero and assign "1" to that point.

Have the children indicate, discuss and assign "2", "3", "4", etc. to points on the line to the right of zero.



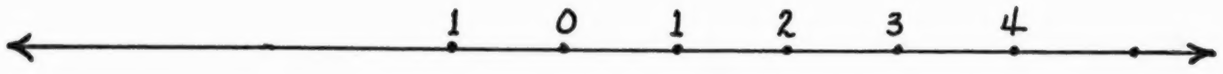
Introduce the Number Line by extending the Number Ray in the other direction.

Ask children to indicate 1 unit to the left of zero.

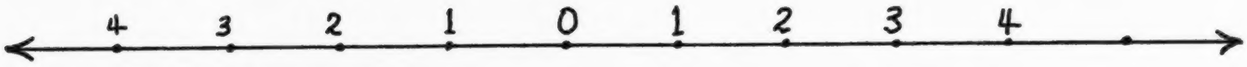


They discuss naming points in this direction to the left of zero.

What would you name the new point? [1]
Label it.



Children should than indicate, discuss and label points 2, 3, 4 to the left of zero.



Have children compare the two points labeled "1".

> What is true of the distance from zero to each of the points labeled "1" ? [same]
>
> What can you tell about the direction of those points from zero? [opposite]

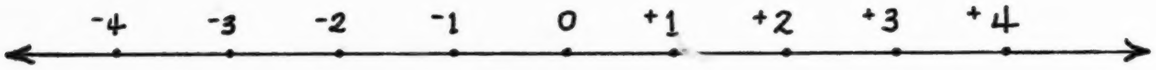
Have children continue to compare pairs of points to discover equal distances from zero; different directions.

3. Develop symbols for positive and negative numbers. Children discuss the need to label these pairs of points on the number line to indicate their differences.

   Ask children:

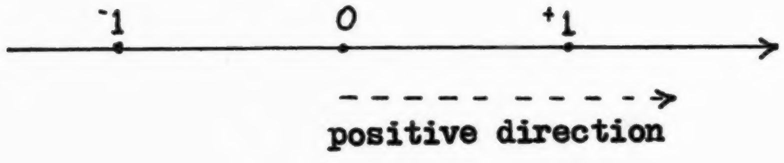
   > If we label points: 1, 2, 3, 4, etc. to right of zero $^{+}1$, $^{+}2$, $^{+}3$, $^{+}4$, what do you think we could label the corresponding points: 1, 2, 3, 4, to the left of zero? [$^{-}1$, $^{-}2$, $^{-}3$, $^{-}4$] Why?

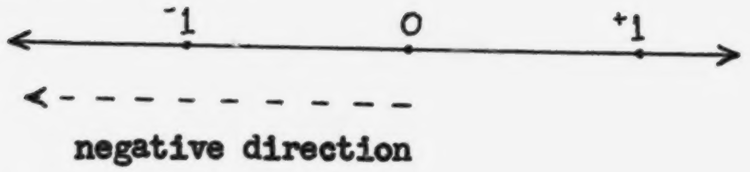
   Children record these symbols on the number line.



Discuss Direction:

> If $^{+}1$ is said to be in a positive direction from zero, how would you name the direction of $^{-}1$? [negative direction]





If the direction of $^+1$ is a positive direction $^+1$ is called positive 1, what would you call $^-1$? [negative 1]

Would you call zero a positive or negative number? [neither]

Discuss pairs of numbers as to direction and distance from zero.

$^+2$ and $^-2$	$^+4$ and $^-1$
$^-3$ and $^+3$	$^-3$ and 5 etc.

Children note that:

$^+2$ is opposite $^-2$ (from zero); $^-2$ is opposite $^+2$ (from zero); etc.

5. Discuss the Whole Number System {0, 1, 2, 3, . . .} with which children have been operating until now.

   Which is the first numeral in the set?
   How many numerals are in the set?
   What other name can be given to 1, 2, . . . [positive 1, etc.]

6. Tell children that this new set of numbers that includes positive and negative numbers and zero is called the Set of Integers.

   Tell children that we sometimes call the Set of Integers

   The Set of Directed Numbers. Why? or
   The Set of Signed Numbers. Why?

7. Children note that:

   The Number Ray maps the Set of Counting Numbers.

   The Number Line is needed to map the Set of Integers.

8. Suggested practice exercises.

   a. Write the symbol which represents the numbers:

negative twenty-six	[$^{-}26$]
positive sixty-four	[$^{+}64$]
negative two hundred seven	[$^{-}207$]

   b. Write the words (number names) for:

   $^{+}15$ $^{-}64$ $^{+}314$ $^{-}708$

   c. If $^{+}6$ represents 6 degrees above zero, what does its opposite $^{-}6$ represent?

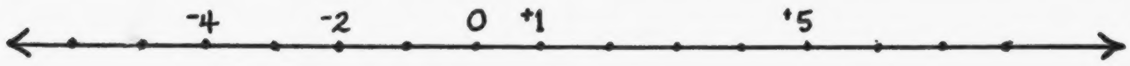
   If $^{+}3$ represents 3 hours after noon, what does its opposite $^{-}3$ represent?

   If $^{+}5$ represents 5 steps to the right, what does its opposite $^{-}5$ represent?

   If $^{-}15$ represents a loss of 15 pounds, what does its opposite $^{+}15$ represent?

   If $^{-}100$ represents a loss of \$100, what does its opposite $^{+}100$ represent?

   d. On the number line below, label the unmarked points.



e. Complete the sequence:

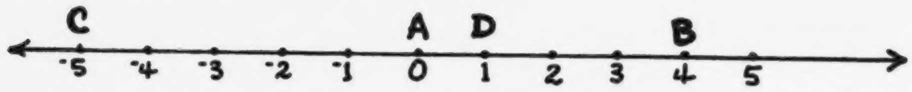
$^{+}2$, $^{+}1$, ___ , $^{-}1$, ___ , $^{-}3$

$^{-}2$, $^{-}4$, $^{-}6$, ___ , ___ , ___

$^{-}6$, $^{-}4$, $^{-}2$, ___ , ___ , ___

$^{-}4$, $^{-}1$, $^{+}2$, ___ , ___ , ___

f. Use the number line to answer the question below.



What is the endpoint of each of the following and what number does it represent?

$\overrightarrow{DC}$	[D, $^{+}1$]
$\overrightarrow{AB}$	[A, 0]
$\overrightarrow{BC}$	[B, $^{+}4$]
$\overrightarrow{CB}$	[C, $^{-}5$]

Making Comparisons

1. Have children draw a number line to compare positive integers in order to determine which is greater, which is less.

   Children note that:

   As we move to the right on the number line, the numbers become greater.

   As we move to the left the numbers become smaller.

2. Extend the number line to the left to include negative integers.

3. Have children explain how the number line shows the following relationships:

$^{+}3 > {}^{+}2$

$^{+}2 < {}^{+}3$

$^{+}3 > 0$

$0 < {}^{+}3$

$^{+}3 > {}^{-}1$

$^{-}1 < {}^{+}3$

$0 > {}^{-}1$

$^{-}1 < 0$

$0 > {}^{-}2$

$^{-}2 < 0$

$0 > {}^{-}4$

$^{-}4 < 0$

$^{-}1 > {}^{-}2$

$^{-}2 < {}^{-}1$

$^{-}3 > {}^{-}4$

$^{-}4 < {}^{-}3$

$^{-}1 > {}^{-}4$

$^{-}4 < {}^{-}1$

4. After many comparisons children note:

Any integer represented on the number line is greater than any integer to its left.

Any integer represented on the number line is less than any integer to its right.

Any positive integer is greater than zero.

Zero is greater than any negative integer.

Any positive integer is greater than any negative integer.

5. Since $^{+}6 > {}^{+}4$ and $^{+}4 > {}^{+}2$, how would you compare $^{+}6$ and $^{+}2$?

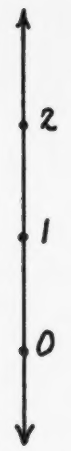
Since $^{-}4 < {}^{-}3$ and $^{-}3 < {}^{-}2$, how would you compare $^{-}4$ and $^{-}2$?

RECTANGULAR COORDINATE SYSTEMS

* UNIT 79 - RECTANGULAR COORDINATE SYSTEMS (Optional)

NOTE TO TEACHER

When children represented numbers on a number line they were dealing with a set of points and a set of numbers. A one-to-one correspondence was set up between each of the sets of points and the number associated with it. By selecting and marking off a scale on a number line any point may be located by its corresponding number on the number line. For example:



When children studied line graphs they were dealing with the elements of two sets and the correspondence between the elements of the sets. For example: In a graph dealing with rainfall, the number of inches of rainfall are matched with dates or with locations.

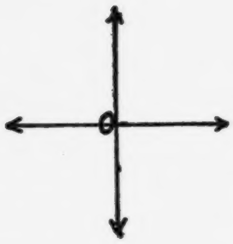
The development has been

1. Points and numbers represented on one line.
2. Inches of rainfall and corresponding dates represented on a grid.

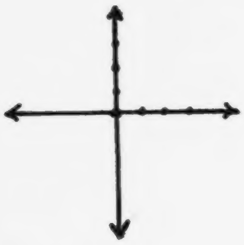
Now we proceed to

3. Coordinates for using an ordered pair of numbers by which any point in a plane may be located.

Two number lines called axes are drawn at right angles and the point of intersection is the zero point of each line.



Both axes are marked off in equal intervals.



To identify a point in the plane of these lines, we assign to it 2 numbers. The first shows its relationship to the horizontal axis; the second shows its relationship to the vertical axis. The horizontal axis is called the X axis. The vertical axis is called the Y axis. The point of intersection O is called the origin.

To locate a point A on the plane, lines are drawn perpendicular to each axis.

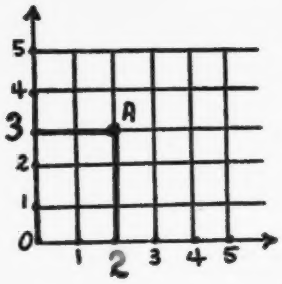


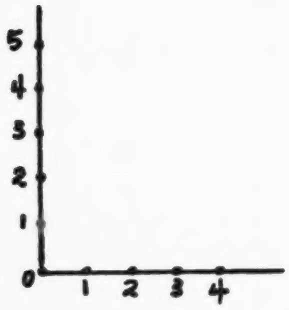
Fig. I

The perpendicular to the horizontal axis in this graph intersects the horizontal axis at the point corresponding to 2.

The perpendicular to the vertical axis intersects the vertical axis at the point corresponding to 3.

2 is called the first or X coordinate.
3 is called the second or Y coordinate.

The coordinate of a point on a number line is defined as the number which corresponds to that point.



Coordinates on the horizontal axis are:
. . ., 0, 1, 2, 3, 4, . . .

Coordinates on the vertical axis are:
. . ., 0, 1, 2, 3, 4, 5, . . .

The location of A in Figure I is described by the ordered pair of numbers (2, 3).

The order of the pair of numbers is of utmost importance.

Suppose the location was listed as (3,2).

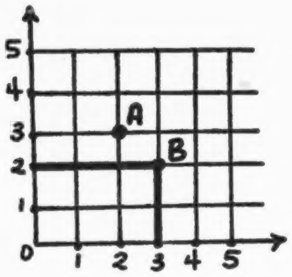
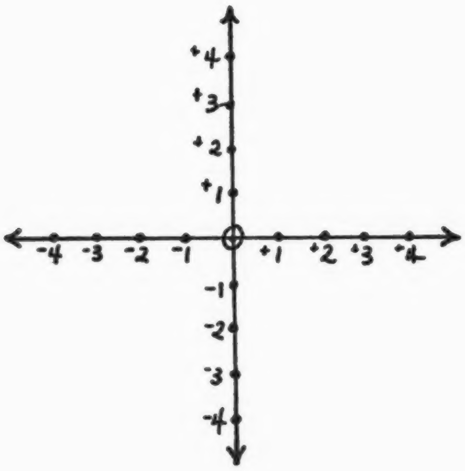


Fig. II

Then a point B, (3,2) shown in Figure II, not the same as point A (2, 3), has been described above.

The first number of an ordered pair is, by convention, the coordinate of the X (horizontal) axis and is called the X-coordinate.
Note that the horizontal and vertical axes are number lines intersecting at zero and including the positive and negative numbers. The direction from zero determines the sign of the number assigned to a point.



It is convenient to use graph paper to help in locating points on a plane since equally spaced horizontal and vertical lines are already printed.

Objectives: To help children understand the graph of a number.

To help children understand the use of an ordered pair of numbers in locating a point on a plane.

To help children understand the graphing of a truth set for an open sentence involving one or two place holders.

TEACHING SUGGESTIONS

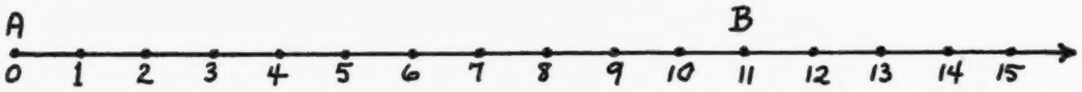
Extending Concepts of Graph to the Graphing of a Number in a Plane

1. Reinforce locating a point on a number line.

Have children:

Observe the number ray below.

Fig. I

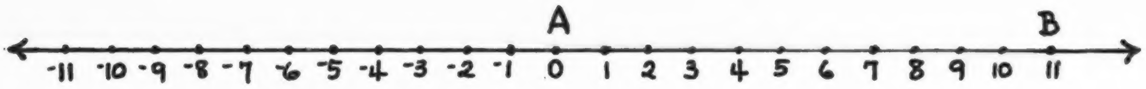

A
B
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Show where point C is on the number ray if the distance of C from A in a positive direction is 3 units.

Mark point C.

Tell how many measurements were required to locate point C.

2. Present a number line such as the one below.


A
B
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10
11

Ask children:

If you know the distance of point C from A and that C is on $\overleftrightarrow{AB}$, how many different points could be named C, if C is 3 units from point A?

[Two points, either in the negative or positive direction from A.]

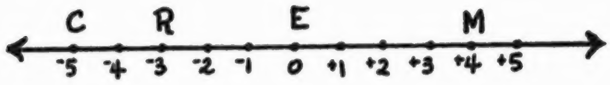
What did we have to know to locate point C in Figure I?

[distance and direction]

What kind of numbers tell both the direction and distance of a point from A. [positive and negative]

Discuss: The number that tells both distance and direction of a point on a line from the "0" point is called the coordinate of the point on the number line.

For example: On the number line below the coordinate of R is $^{-}3$; the coordinate of M is $^{+}4$.



3. Tell children that each point in a number line can be called the graph of the number to which it corresponds. If we wish to consider the graph of a set of points it helps to darken the points corresponding to their numbers.

For example:

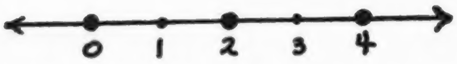


Fig. II

Figure II is the graph of the points whose coordinates are 0, 2, and 4, or the set { 0, 2, 4 }.

Ask children to graph the following sets of points whose coordinates are:

$^{-}3$ 0 4 9

Graph each on a separate horizontal and vertical number line.

Present a vertical number line.

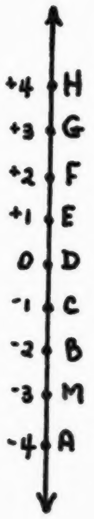
Direct children:

Show the graph of 2.

Circle the coordinate of point B.

Graph the solution set for

n = 0 + 4 on another number line



Locating Points In a Plane

1. Ask children to observe the number line and points R and S below and answer the questions.

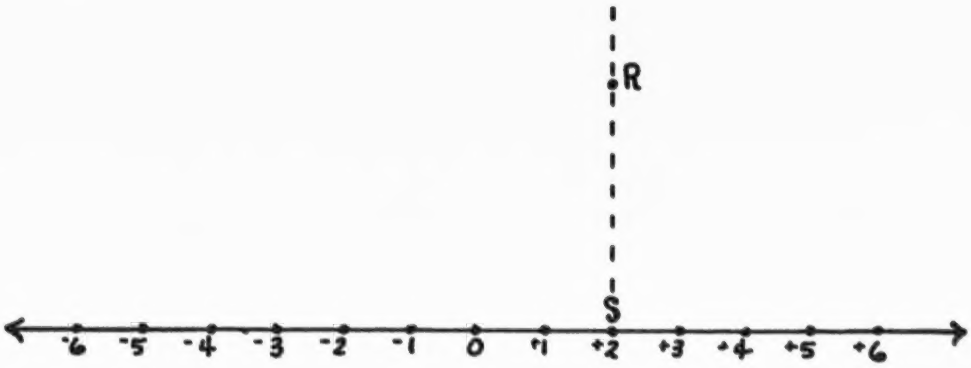


Fig. I

What is the coordinate of point S?
How can we state the position of point S?
Can we tell the direction of point R from point S?
Can we state the position of point R? Why or why not?

[No, we do not know how far above the line it is.]

2. Tell children that we can state its position by using a second (vertical) number line which intersects the horizontal number line forming right angles and which has the same zero point.

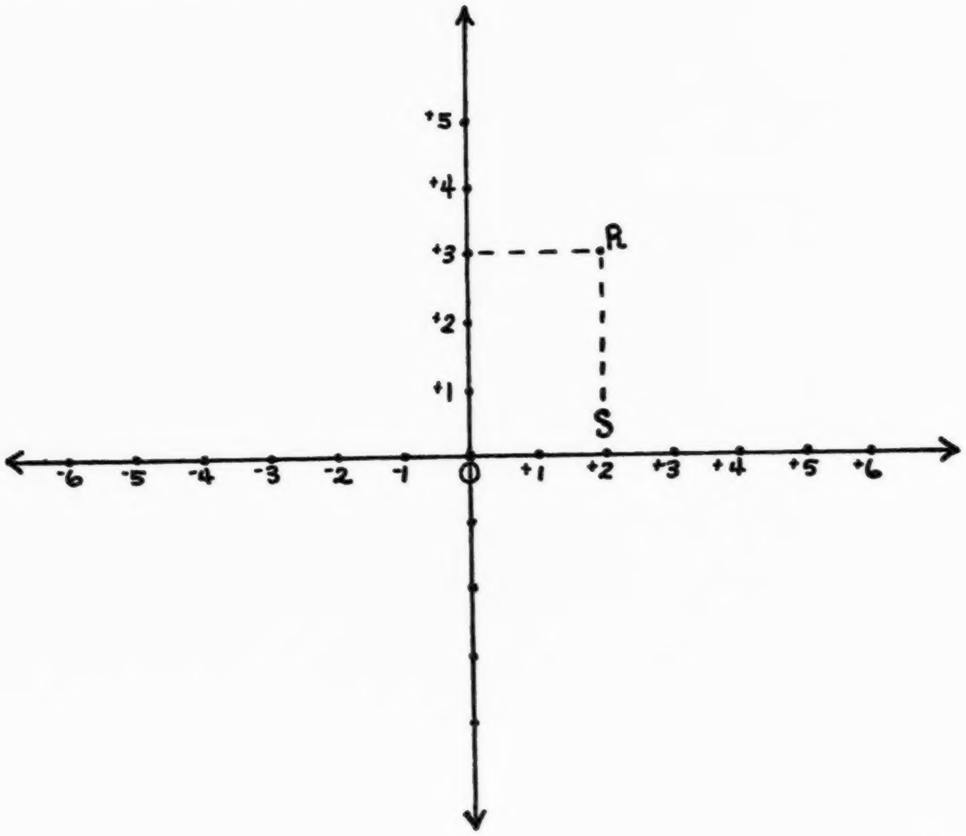


Fig.II

Children should compare Figure I and Figure II.

Direct children:

How did we describe the position of point R in Figure I?
[above $^{+}2$]

Observe Figure II. Can we state the position of point R more exactly now?

[Yes, it is above $^{+}2$ on the horizontal axis and to the right of $^{+}3$ on the vertical axis.]

How do we describe direction on a number line?

[positive direction
negative direction]

Is there any point except R which is exactly above $^+2$ on the horizontal number line and also exactly to the right of $^+3$ on the vertical number line?

[NO]

How many numbers are required to describe the position of point R?

[2]

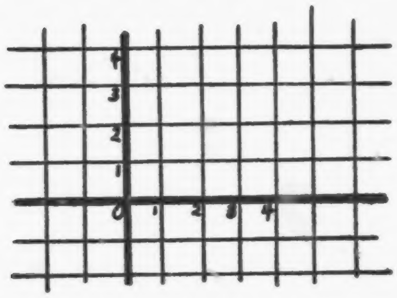
3. Tell children:

The two numbers necessary to describe position in a plane form an ordered pair of numbers called the coordinates of the point.

The first number of an ordered pair tells the number on the horizontal axis; the second number of the ordered pair tells the number on the vertical axis.

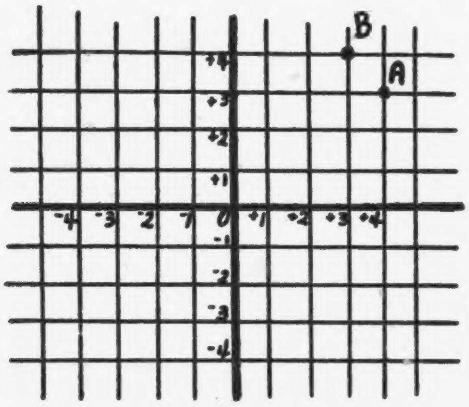
We write the ordered pair for R as: (2, 3).

4. Discuss use of graph paper to help in locating the position of points in a plane. (The lines are already drawn perpendicular.)



The Importance of Ordered Pairs

Have children examine a graph such as the one below:



Ask children:

What are the coordinates of A? $[\ (^+4, ^+3)\ ]$

Which coordinate is the horizontal coordinate; the vertical coordinate?

What are the coordinates of B? $(^+3, ^+4)$

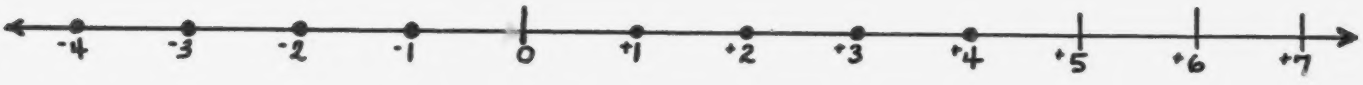
Which coordinate is the horizontal coordinate; the vertical coordinate?

Do the coordinates $(^+4, ^+3)$ and $(^+3, ^+4)$ locate the same point? Explain.

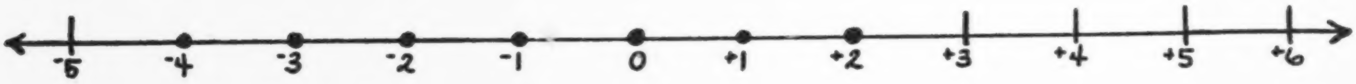
Teacher should emphasize the importance of "ordered pair".

Suggested Practice Exercises

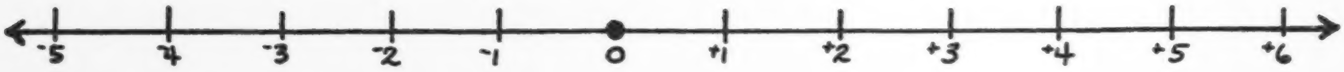
1. List or describe the sets of integers whose graphs are shown below.



$[\{^-4, ^-3, ^-2, ^-1, ^+1, ^+2, ^+3, ^+4\}]$



$[\{ {}^-4, {}^-3, {}^-2, {}^-1, 0, {}^+1, {}^+2\}]$



$[\{0\}]$

2. Draw two number lines showing the integers from ${}^-5$ to ${}^+5$ and graph the sets below; first on a horizontal number line and then on a vertical number line.

   a. $\{ {}^-4, {}^-2, {}^+1, {}^+3 \}$

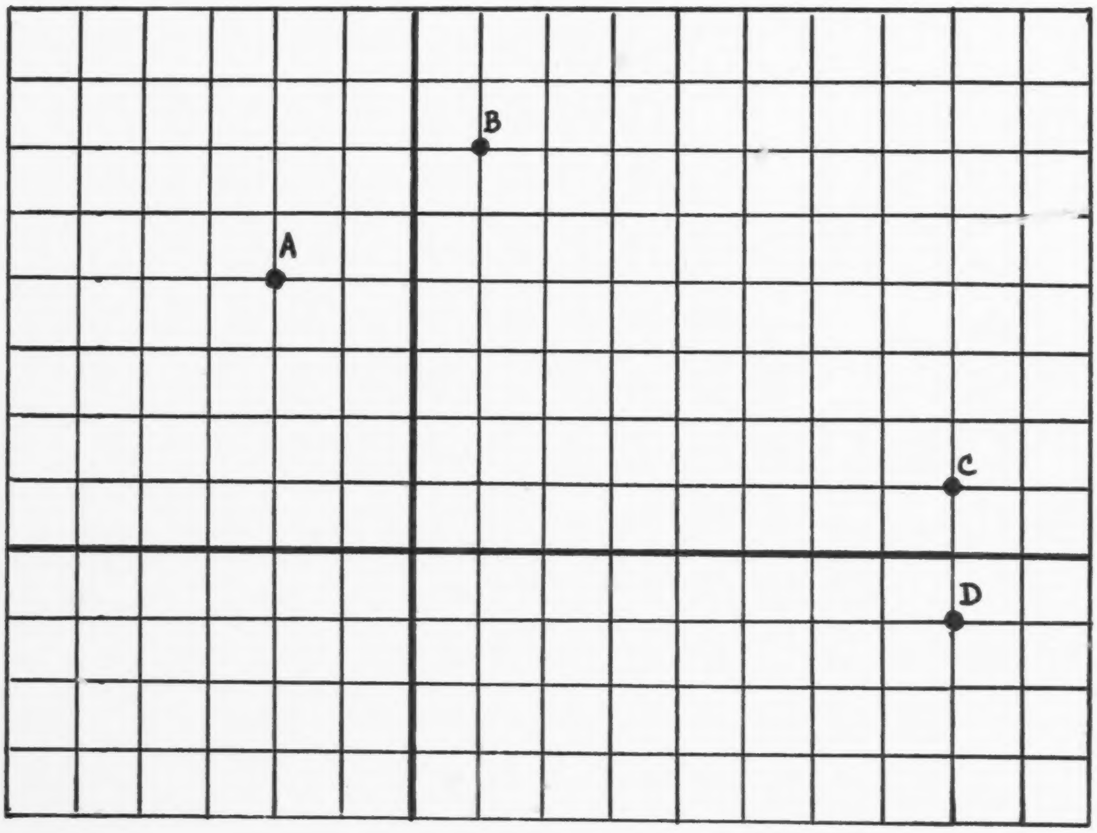
   b. $\{ {}^-3, 0, {}^+3, {}^+4, {}^+5 \}$

3. Use graph paper. Choose two perpendicular lines for coordinate axes and darken them to show the lines chosen.

   Graph the following ordered pairs and label each point.

   A $({}^+2, {}^+5)$ B $({}^-1, {}^+4)$ C $({}^+3, {}^-2)$

4. Write the coordinates (ordered pairs) for each labeled point on the graph below.



5. Locate four points forming a square on the above set of axes and indicate the coordinates of each point.

6. Do the same (as ex. 5) for other geometric figures.

Graphing the Truth Set For An Open Sentence. (Reference: Student's Discussion Guide - Madison Project - pp 18, 19)

1. Present an open sentence; e.g., $\square + \triangle = 5$

Have children find the solution set when the replacement set is {0, 1, 2, 3, 4, 5}.

(Review meaning of replacement set.)

As children respond, teacher may tabulate as follows:

□ + △ = 5

When □ = 0 then △ = 5
When □ = 1 then △ = 4
etc.

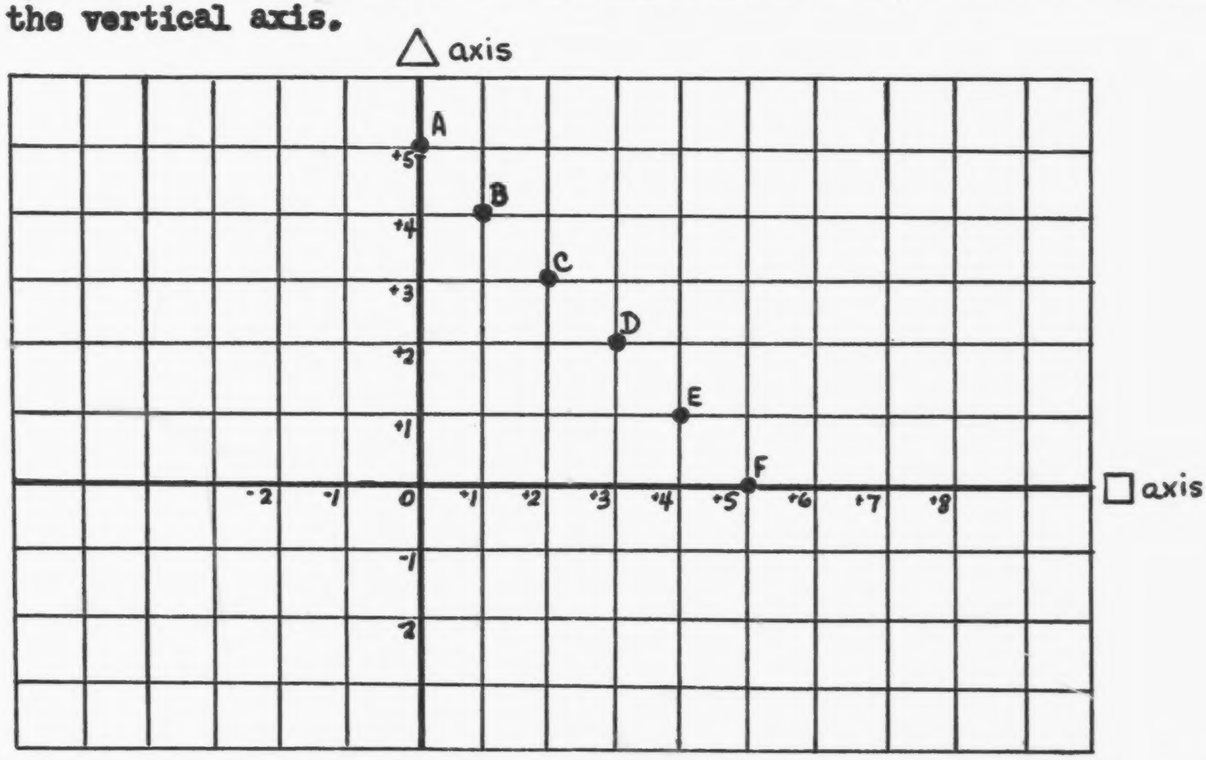
□	△
0	5
1	4
2	3
3	2
4	1
5	0

Ask children:

How many ordered pairs will make the sentence true?

What are the ordered pairs? [0, 5; 1, 4; 2, 3; etc.]

Have children graph each of the ordered pairs in the truth set. They may place the numbers for □ on the horizontal axis, numbers for △ on the vertical axis.



Have children observe that these points lie on a straight line.

*2. Have children explore the same problem when the replacement set includes negative numbers.

For example: if the replacement set is $\{ {}^{-}3, {}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, {}^{+}3 \}$

3. Have each child prepare a table to represent the truth set for the following open sentence:

$\triangle = \square + 1$

For example:
When $\square = 0, \triangle = 1$
$\square = 1, \triangle = 2$

$\square$	$\triangle$
0	1
1	2
2	3

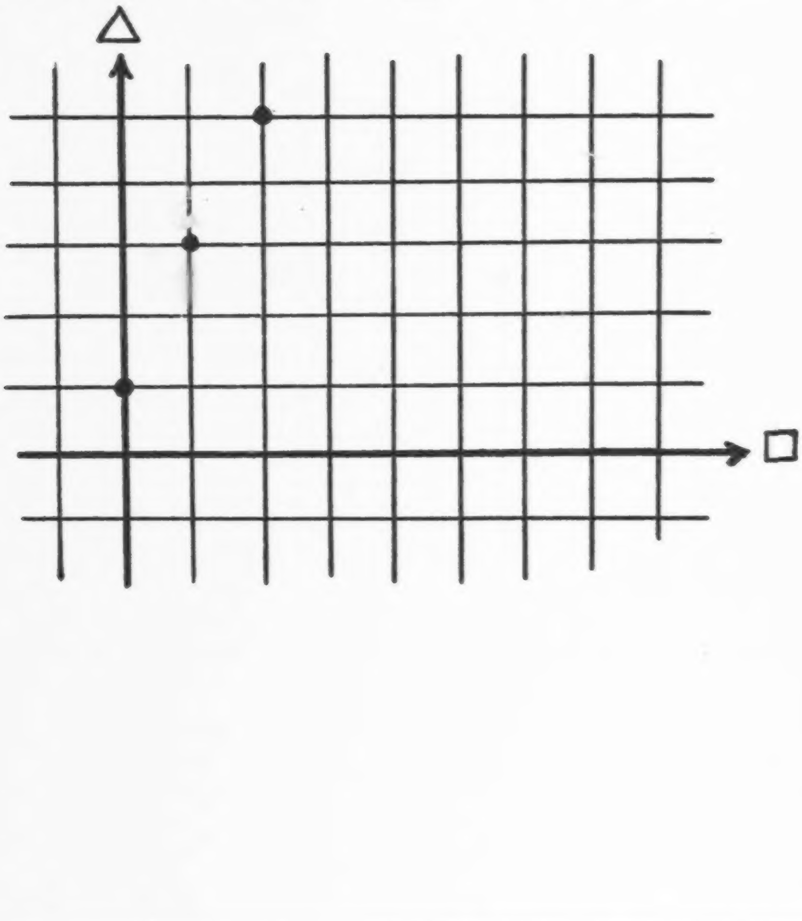
etc.

4. Have children graph the truth set for $\triangle = \square + 1$ when the replacement set is $\{4, 5, 6, 7\}$.

Suggest to children that $\square$ represents the horizontal axis, $\triangle$ represents the vertical axis.

Observe that the points of the solution set lie on a straight line.

5. Present a graph as follows. Tell children that the graph has been started for the open sentence $\triangle = (2 \times \square) + 1$



Have children mark 3 more points on the graph for the open sentence above.

Ask children to "guess" another point, to explain why they picked it, and then to verify their "guess" by substituting the value for □ and △ in the open sentence.

Suggested Practice Exercises

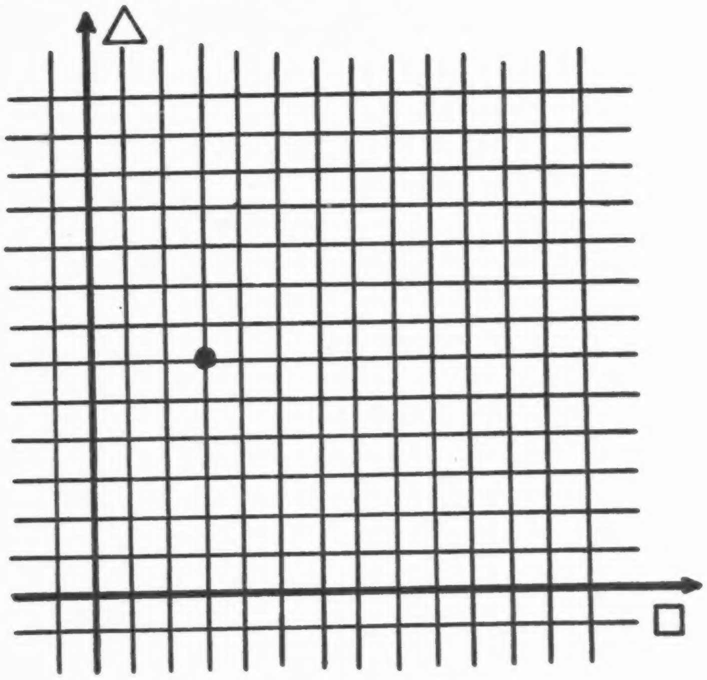
1. Use the table to mark 4 more points on the graph below.
(Reference: Madison Project - Student's Guide - p. 18)

△ = □ + 3

Open Sentence

□	△
3	6
4	7
5	8
6	9

Table for Truth Set



Graph for Truth Set

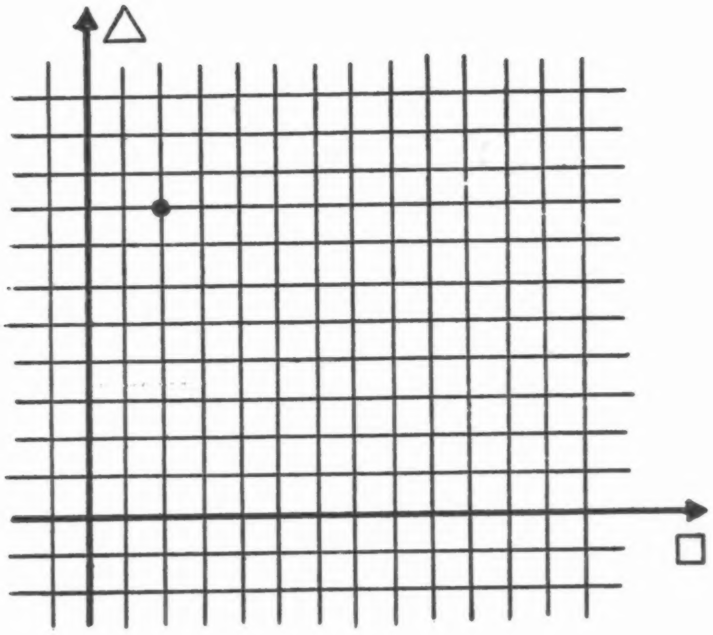
2. Complete the following table to represent the truth set for the open sentence: △ = (4 x □) + 2

□	△
2	4
3	
4	
5	

Children graph the truth set.

Children may connect the points by drawing line segments between them. What do they observe?

3. Write at least 3 true sentences represented by the graph below.



$\triangle 8 = \boxed{2} \times 4$

$\triangle 8 = \boxed{2} + 6$

etc.

*UNIT 80 - PROBABILITY (Optional)

NOTE TO TEACHER

Why Probability?

Much that occurs in our lives depends upon chance. Part of our lives is spent considering uncertainties. Things may happen or they may not happen.

Probability concepts and calculations have become increasingly important in our modern age. Medicare, business, weather, insurance, investments all involve consideration of a probability measure. Hence its introduction in its simplest forms into the elementary school curriculum is desirable.

A few illustrations will give us a clue to some simple applications of the mathematics of Probability. If a die (singular of dice) is thrown what are the chances that a 2 will appear? Since there are 6 possible numbers that may appear (a cube has six faces, hence 6 outcomes) and since it is equally likely that any one face may turn up, there is one out of 6 that a 2 will turn up.

If in the dark I reach into a drawer containing 6 gloves, paired and placed in plastic bags, 2 blacks, 2 white, 2 brown, what are my chances of extracting a pair of a given color? The chances are 1 to 3 of finding a pair of brown gloves. ($\frac{1}{3}$ would not be the answer if the 6 gloves were loose in the drawer.)

The Probability of an event may be defined as the ratio of the number of favorable ways that the event can occur to the total number of equally likely outcomes.

To introduce concepts involving probability, experiments should be performed and a count made of the outcomes. For example:

1. Toss a coin many times and observe the number of times a head appears.

2. Toss two coins and observe the number of times
    a. 2 heads
    b. one head and one tail
    c. 2 tails

   come up.

3. Toss a cork from a bottle 5 times.

4. Other experiments involving similar situations.

If children will keep a tally of the number of times a given outcome or set of outcomes (called an Event) occurs and the total number of times the experiment is performed, the ratios involved should prove interesting. For example: In (1) the ratio 1 : 2 (or fraction $\frac{1}{2}$) will show up after many tosses of the coin.

In 50 tosses the likelihood is that heads will turn up about 25 times; in 100 tosses the chances are that heads will turn up about 50 times, etc.

Objectives: To experiment with "What are the Chances".

To introduce beginning concepts of Probability.

TEACHING SUGGESTIONS

Suggested Experimentation

Tossing A Coin

1. Ask children to toss a coin.

When one coin is tossed what is the number of possible ways that the coin can fall? What is the chance of heads turning up out of the 2 possibilities?

[1 chance out of 2 possibilities]

Tell children that another way of saying this is the probability is 1 out of 2 or $\frac{1}{2}$.

2. Have children record results to see how probabilities change when the same action is repeated.

We know that in tossing a coin once the probability is "1 out of 2" that heads will turn up, or that tails will turn up. If we toss the coin a second time, then a third time, etc., what will happen?

Have children keep a tally. For example: The table below shows the results one child may have in 8 tries:

H	T
///	/////

Ask children:

In examining the Tally of Ellen's experiment:

1. How do you know the number of tries? [count]
   How many were there? [8]
   How many times did heads turn up? [3]
   How many times did tails turn up? [5]

   After 8 tries what is the probability that a head will turn up on the 9th try?

3. Have children extend their tallies by experimenting to see what happens after 10 throws, 20 throws, 30 throws, 50 throws.

   Children should experiment as many as 100 times.

4. Have each child compare his tally with the tallies of other children.

Children should observe that in many tosses, a coin may be expected to come down heads as many times as it comes down tails. It does not mean that a coin will come down alternately heads and tails in consecutive throws.

5. Experiment with an irregular object, such as a cork of a bottle or a paper fastener, where there are two possible ways it could fall. Find the Probability of the object falling one way by a tabulation.

6. Have children tabulate for the following experiment:

   If Alan picked a ball from a box, containing blue and red balls, noted the color, put it back and picked again, and did this 30 times, about how many times should he expect a blue ball to be picked? A red ball to be picked?

7. Further Experimentation

   Some children may wish to experiment keeping tallies of the probabilities when tossing 2 coins; 3 coins; etc.

   For example: In one person's tally, out of 16 throws of 2 coins, heads turned up 4 times, head and tail turned up 9 times, etc.

0 Tails 2 Heads	1 Tail 1 Head	0 Heads 2 Tails
////	///// ////	///

Children should make and tally enough throws to make a prediction.

Have children compare the actual results with the predicted possibilities.

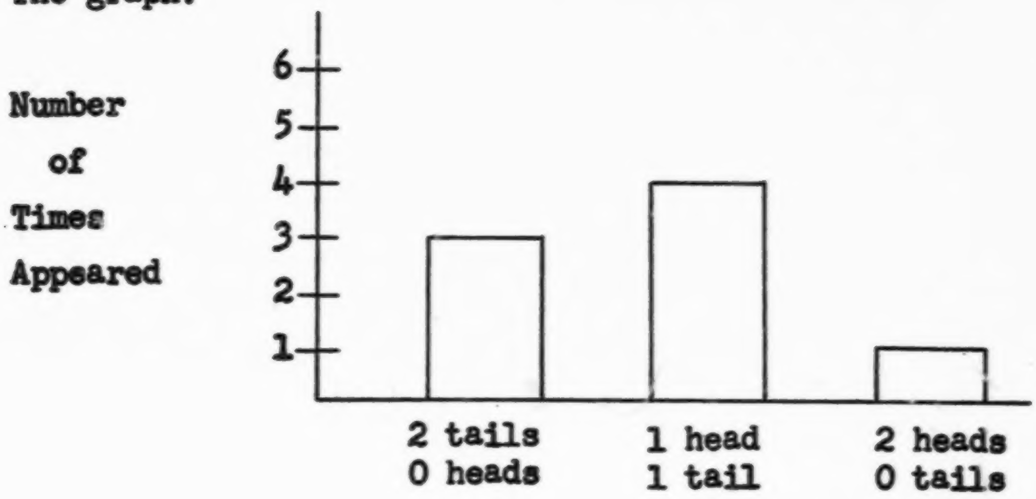
Graphing Probabilities

Have children make a bar graph of the results of their tallies.

a. For 1 child's tally of 8 throws of 2 coins, with the result of:

2 heads, 0 tails — 3 Times
1 head, 1 tail — 4 Times
0 heads, 2 tails — 1 Time

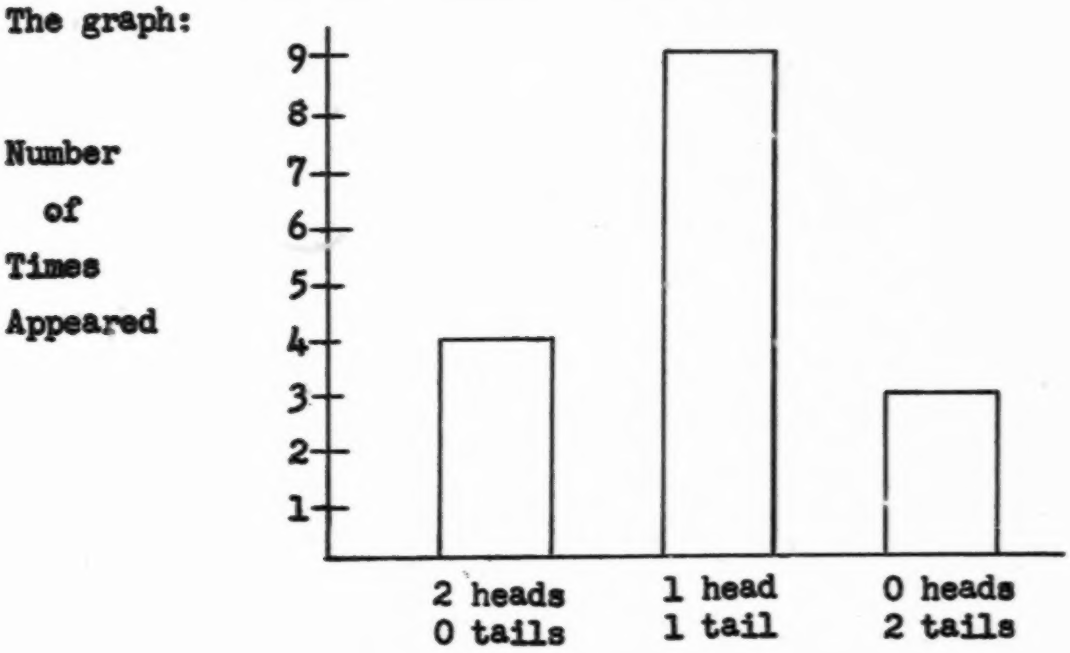
The graph:



b. Have the child graph the result of 16 throws. The result might be as in the tally above:

2 heads, 0 tails — 4 times
1 head, 1 tail — 9 times
0 heads, 2 tails — 3 times

The graph:



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c. Children should graph the results of many experiments e.g., 20 throws, 40 throws, etc.

Discuss which possibility is most likely to occur.

Have children compare the various graphs to see the shape of the curve that evolves as the number of tossings increases indefinitely.